WORD PROBLEMS ON MULTIPLICATION AND DIVISION: A LITERATURE REVIEW

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Introduction

Arithmetic is the oldest branch of mathematics which deals with numbers, operations on them, and the number system. The four basic operations are addition, subtraction, multiplication and division. Students seem to quickly grasp the concept of putting together things (i.e. addition) or taking away things from a group (i.e., subtraction). However, the concept of multiplication and division seems difficult for the students to understand.

Word problems involving these two operations (multiplication and division) add to the difficulty faced by students while learning the subject of arithmetic. This paper is an attempt to understand the reason that makes it difficult for students to solve word problems on multiplication and division by looking at the different kinds of questions involved in multiplication and division that a teacher should be aware of. It also attempts to look at the misconceptions a child develops in her early years while learning the concept and the ways in which a teacher can take care of these.

Difficulties with Multiplication and Division

Mulligan and Mitchelmore in their research found that counting strategies were integrated into repeated addition and subtraction processes and then generalised as multiplication and division. Students begin by practicing multiplication and division with many different numbers and gradually take this procedural knowledge to apply conceptually in the form of word problems (Mulligan and Mitchelmore 1998).

Word problems are important for two reasons. First, these help in identifying various types of difficulties children encounter when they begin to work on mathematical problems. Second, these help the students analyse complex problems, whether or not they build on the same theoretical ideas used to analyse simple problems (Reed 1999).

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A child may be able to solve operations as such, but in a word problem, she has to do more than just the simple operations. It involves interpreting the question to identify the relevant applications of operations. It is through a word problem that a child actually learns to apply the operations he has learnt so far, thus demonstrating his understanding of these. Also, any two word problems could be based on the same operations and methods. But the ease and comfort of a child to understand these and perform the relevant and necessary operations in one question than the other (as the difficulty level could be different for the two questions) gives an understanding of the scaffolding that needs to be constructed to make the child efficient in solving the problems.

Word problems involve not just conceptual understanding of a topic but also the ability of a child to comprehend the language in order to identify the correct operations to use. I asked many of my friends if they had any difficulty in doing word problems and a common response was they would take a lot of time in just understanding the language so as to proceed with the question. Hence, they struggled considerably with word problems. It is this confusion along with the lack of clarity of concepts that leads to difficulty in interpreting the operations for word problems.

Unlike addition and subtraction, word problems involving multiplication and division involve referring to many objects. Not just that, the objects specified in the answer or solution part of the question could be very different from what has been mentioned in the question itself (Reed 1999).

An example would be – *Ram wants to buy 5* ***pens****. If* ***cost*** *of one is Rs.10, how much* ***money*** *he has to pay?* as opposed to an addition problem which would have been *Ram has 5* ***pens*** *and his brother has 4* ***pens****. How many* ***pens*** *do they have in all?*

Another reason that makes these two operations slightly difficult than addition and subtraction is that these require children to develop a multiplicative reasoning which involves different forms and different meaning making situations. Three major kinds of multiplicative reasoning have been identified as: (a) one to many correspondence (b) co-variation and (c) sharing, division and splitting (Bryant 1996).

One to many correspondence: A child should be able to establish relationship between two sets of objects. For example, she should be able to relate one car with four wheels. This group of one car and four wheels together form a composite (single unit). Another concept of ratio is introduced here indicating that if there are two cars the number of wheels would become eight and so on (to maintain the ratio 1:4). This process of adding a certain amount to maintain the one-to-many correspondence is termed as replicating. A child needs to understand this as well as the inverse of it i.e., if one car is removed then 4 wheels would be removed.

Co-variation: In this reasoning, children should be able to understand the idea of 'rate`. For example, if sugar`s weight and its cost are to be connected, it is done by a third variable which is 'cost per kg` (called intensive quantities). Introduction of this new variable could be confusing for the children as it is neither the actual cost nor the actual weight but just a relationship between the two. If the weight and cost are increased even then this intensive quantity (cost per kg) has to remain constant.

Sharing, division and splitting: This is also termed as part-whole model. In this, an understanding of whole (complete) and part (something that is taken away from whole) is being developed in children. Teachers need to build the understanding that if the number of equal parts and the size of each part is known, multiplication should be performed to find the whole. On the other hand, if the whole and either the number of equal parts or the size of each part is known, division should be performed to find the solution (Huinker 1989).

This idea is explained later in this paper as a part-whole concept.

Joanne T. Mulligan (1997) in his paper, 'A research-based framework for assessing early multiplication and division strategies`, classifies six stages of developing an understanding about multiplication and division in children:

1. Initial Grouping and Perceptual Counting: counts each item as one (perceptual) as the children are unable to see equal groups as composite units.

2. Intermediate Composite Groups: counts equal groups using rhythmic, skip or double counting; if the items are visible in the groups then count the number of groups and number in each item.

3. Abstract Composite Units: counts composites even when they are not visible.

4. Repeated additions and repeated subtraction: co-ordinates composite units and uses composite units, a specific number of times, as a unit, e.g. : 2+2+2

5. Multiplication and division as operations: able to co-ordinate two composite units, e.g: three groups of four make 12.

6. Known multiplication and division fact strategies: able to recall the facts related to the operations, use multiplication and division as inverse relationship.

There is a variety in the type of word problems in multiplication and division a child has to deal with (Mulligan 1992):

(a) Multiplication:

1. Repeated addition: Example - There are two tables in the classroom and four children are seated at each table. How many children are there altogether?
2. Rate: Example- Ram wants to buy four plastic cars. Cost of one such car is Rs. 50. How much does he have to pay?
3. Factor: Example - I have three times as much money as my brother. He has Rs 9. How much money do I have?
4. Array: Example- There are four lines of children with three children in each line. How many children are there altogether?

(b) Division:

1. Partition (Sharing – to determine the size of each part): Example- There are eight children and two tables in a classroom. How many children can be seated at each table?
2. Rate: Example- Sam bought five chocolates for Rs.60. How much does one chocolate cost?
3. Factor: Example- I have Rs 27 which is three times what my brother has. How much money does my brother have?
4. Quotition: (Also known as measurement model. It shows the number of parts that are there in a whole): Example- There are eight children and four of them are seated together at a table. How many tables are there in the classroom?
5. Sub-division: Example- I have three apples to be shared evenly between six people. How much apple will each person get?

Intuitive Model

Several research studies conducted over a period of time suggest that children are able to perform operations even before they enter a formal education set-up. Thus, they come with their own informal or intuitive strategies to solve such word problems. Educators need to make a connection between these intuitive strategies with the formal methodology being taught in school. In the absence of such a connection there is a shift from intuitive and meaningful problem solving approaches to mechanical and meaningless ones (Mulligan 1992). One such research study conducted over a period of two years indicates that 75% of the 70 sample (7-8 years olds selected from eight Catholic schools in the Sydney Metropolitan Area) students were able to solve multiplication and division word problems even though they had not received any formal instructions (ibid). For approaching division questions, children used sharing one-by-one, building up (additive) or building-down (subtractive) model. This study showed that for division problems in the category of partition, quotition and rate, children would try to 'build-up` the divisor to reach the dividend so as to arrive at the answer. For example, if a question involves the operation 16/2 to be applied, children would form groups of 2 such as 2, 4, 6....till 16 is reached (also demonstrating knowledge of skip-counting). For the same question they would also approach it in a build-down method, ie, starting from 16 and removing 2 till 0 is reached. Building-down method was more common in students and only when they were familiar with additive and multiplicative strategies were they able to use building-up model (ibid).

The various solution strategies adopted by the students in this study was categorised as follows: a) direct modelling with counting (counting-all, skip or double counting); and b) no direct modelling, with counting, additive or subtractive strategies, and the use of known or derived facts (addition, multiplication). From this study it can be concluded that there is a need on the teacher`s part to be able to help the students build relationships between their intuitive models and the formal strategies taught at schools.

Misconceptions

Another research study shows that children do not understand the concepts to identify the correct operations for the word problem but instead rely on the numbers to arrive at an answer. Consider the following interaction between an interviewer (I) and a student Ann (A) of grade six (Reed 1999):

I: A bag of snack food has four vitamins and weighs 228 grams. How many grams of snack food are in six bags?

A: Probably divide that.

I: Why division?

A: Because there`s a big number and a small number.

A: (After dividing 228 by 6 and obtaining 38) no...times (multiply).

I: Why are you changing your mind now?

A: Because 38 is much less than that one (228) and that`s only in one bag. That makes better sense.

I: What can you tell students to help them?

A: if you see a big number and a small number, go for the division. If that doesn`t work, then you try other ones.

From this study, it was concluded that students:

a. Use arithmetic operations they feel most competent with or that has been recently discussed in the class.

b. Use the size of numbers to arrive at an operation.

c. Try all operations and choose the most reasonable answer.

d. Look for key words (Such as 'all together` for addition, 'less` for subtraction)

Such are the misconceptions with which a child keeps progressing to higher grades, and, thus, find himself less confident when it comes to solving word problems. Even as a student and a teacher myself, I remember marking the 'key words` to identify the operation to be performed. However, this recognition of key word would not always give the correct answer. Take, for example, the following question- *There are two tables in the classroom and four children are seated at each table. How many children are there altogether?* The key word 'altogether` would immediately make the children do addition even though the correct operation here is multiplication. Giving the key-words looks like a helping aid but many good teachers do not recognise the harm it may be end up doing.

Another major misconception that students have is that multiplication always makes bigger and division makes smaller. However, this is not true when a problem involves rational numbers, fractions, mixed fractions or decimal numbers. This misunderstanding is enhanced if students` only understanding of multiplication is that of repeated addition. Consider the following example (Campbell 1993):

If a car travels 30 miles on one gallon of gasoline, how far will it travel on ½ gallon?

Based on the reasoning that value of a lesser quantity (1/2) has to be found out, students tend to apply division here instead of multiplication. Hence teachers need to expose students to a variety of problems which could help them understand that multiplication does not always make it bigger. Students are also hesitant to accept that the multiplication of two numbers can lead to a smaller number (i.e., 0.5*0.3=0.15). For this, a suggested approach that the teacher should use is the area model to explain the idea that the product of two numbers less than one is a number smaller than either of the two numbers. In the area model, one factor is the length of a rectangle and other factor is the width of it. The product then is determined by the area of this rectangle. Only when students are comfortable using this model with whole numbers should the teacher then extend it to explain multiplication of decimal numbers, fractions and rational numbers.

The same study also shows that for division a measurement interpretation (i.e., when a total is given and the size of each set is known to the children. They are asked to find out how many such sets can be made) should be used. These meanings should be constructed in some specific context as children cannot gather much from an isolated question such as what is 2 / 0.25. A question such as - 'How many 0.25 s are in 2?` - would help the children understand that division does not necessarily have to make it smaller.

Teachers need to develop strategies for children that will force them to think about a reasonable solution. When a student of grade four was asked to form a question for the statement 3*6, his response was- I have three cars. I gave six cars to my brother. How many do I have now? And for this he claimed that the answer is 18 (Huinker 1989). The paper by Huinker (1989) suggests a strategy that may be productively adopted by teachers to develop this thinking ability in the students. He followed the same method with grade three students and could see a huge difference in their understanding of word problems related to multiplication and division. This method involved establishing a part-whole (sharing, division, splitting) relationship. Based on this knowledge, children then solved certain questions to demonstrate their understanding. For this children need to form an understanding that anything in total is the 'whole` and something out of that whole forms a 'part` of it. This idea was established using regular sentences such as: *the whole class may go out for recess, there aren't enough chairs for whole class, part of the students will sit on the floor.* A misconception that needs to be avoided here is that children do not consider the parts as equal parts, and, thus, are not able to arrive at a correct solution. This idea of 'equal parts` should be emphasised by the teacher. Then the understanding needs to be built in the students that when parts are given (number of parts and number in each part), to find the whole, multiplication needs to be done. For example in question such as- Four friends (i.e., number of parts) *went shopping. They each bought two tapes* (i.e., number in each part). *How many tapes did they buy* (i.e., whole)?

For the above question, before explaining the part-whole model, 62% students were able to give the correct response which post instruction rose up to 84%. In an inverse manner, when the whole is given and either the number of part or the number in each part is to be calculated, division is used. For example, Sarah had twelve stickers (ie, whole). She gave them to four friends (i.e., number of parts). She gave the same number of stickers to each friend (equal parts). How many stickers did she give to each friend (ie, number in each part)? (ibid)

This method was also found useful in explaining the comparison problems such as – *(i) Kim read two pages in her book. Juan read three times as many pages as Kim read. How many pages did Juan read in his book* (involves multiplication)? *(ii) Jessica has eight jelly beans. Shawna has two jelly beans. Jessica has how many times as many jelly beans as Shawna* (involves division)? (ibid)

However this multiplicative reasoning has its own complexities. For instance, now children have to deal with three groups: the whole, number of parts and number in each part. An understanding has to be developed that if the whole (for example, total number of sweets) remain same and the number of parts (for example, the number of children) are increased, then the number in each part (number of sweets each child gets) will be decreased. But if the number of sweets is increased and the number of children remains constant, then sweets per child would increase. Such new and varying relationships are difficult for children to understand (Bryant 1996).

Conclusion

Understanding the variety of complexities present in the concepts of multiplication and division by teachers and educators has a huge implication for them to be better prepared while introducing the concept to their students. Teachers need to be more aware of the fundamental problems children face while dealing with multiplication and division questions. They should check for the reasoning behind a solution instead of just focussing on the final answer. Otherwise, the misconceptions are carried forward to adulthood and people find it difficult to get rid of these (Campbell 1993). Teachers should probe in detail so as to focus on appropriate structural aspects rather than on specific details (such as key words or numbers) of the problem. It is important that students understand the structure of a problem and not solely depend on key words to solve a word problem. The idea of multiplication and division could be made interesting by the teachers by using manipulative and visual aids. Such a visual representation of a problem helps a child to develop a better understanding and enables her to make connections with the intuitive knowledge that she already has. The language and the context in which problems are situated in, play important roles for children to be able to interpret and represent the problem. Therefore, keeping this in mind, teachers should be able to scaffold the problem for the child so that she is able to understand it better.

References

Bryant, T. N. (1996): *Children Doing Mathematics* (Oxford: Blackwell Publishers Ltd).

Campbell, A. O. (1993): "Misconceptions about Multiplication and Division", *National Council of Teachers of Mathematics:* 408-411.

Huinker, D. M. (1989): "Multiplication and Division Word Problems: Improving Students` Understanding", *The Arithmetic Teacher,* 37(2): 8.

Mulligan, J. (1992): "Children`s Solution To Multiplication and Division Word Problems: A Longitudinal Study", *Mathematics Education Research Journal,* 4(1): 24-41.

Mulligan, J. (1998). "A research-based framework for assessing early multiplication and division". In *Proceedings of the 21st Annual Conference of the Mathematics Education Research Group of Australasia,* Vol. 2: 404-11.

Reed, S. K. (1999): *Word Problems- Research and Curriculum Reform* (Routledge).

Valentin, J. D., and Chap-Sam, L. (2005). "Roles of semantic structure of arithmetic word problems on pupils` ability to identify the correct operation", *International Journal for Mathematics Teaching and Learning* (electronic journal).

Appendix I: Sample questions from NCERT textbooks (Primary Grades)

Grade 3

- The almirah has 4 shelves. There are 5 books in each shelf. How many books are in the almirah? (repeated addition type – find out whole)
- I have 20 books. I can keep 5 books in one shelf , so how many shelves do I need in my almirah?
- Share 25 bananas among 5 monkeys. How many bananas for each monkey? (partition/sharing type – find number in each part)

Grade 4

- There are 210 students in a school. If there are 4 buses, how many children will get seats? (partition)
- There are 35 students in 7 rows. Each row has how many students? (partition)
- Meera made 204 candles to sell in the market. She makes packets of 6. How many packets will she make?

Grade 5

- We buy fresh fish for Rs15 per kg. For 6 kg fresh fish, how much we`ll have to pay? (rate)
- One kg of tomato cost Rs.12. How much 2 kg of tomato would cost? (rate)
- 352 children from a school went on a camping. Each tent had 4 children. How many tents do they need?