

Thoughts on the Division Operation

Notes from a Reader

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Among the four basic arithmetical operations, division is easily the most complex, and it is the one with which children have the most difficulty. In this regard, I would like to share the following thoughts with readers on how division can be taught. They may be regarded as alternatives to the way division was presented in the pullout of an earlier issue. In particular, the following questions have been addressed:

1. Why do we proceed from left to right in division whereas all the other operations proceed from right to left?
2. What is the role of place value in the standard division algorithm?
3. How do we estimate the quotient when the divisor has two or more digits?

Points 1 and 2 are connected. The argument I use with students is that it is just a matter of efficiency. In the case of addition and subtraction, a left to right algorithm will not be efficient, as illustrated below.

Left to right					Right to left				
Step 1		T	U	We start by adding the tens.	Step 1		T	U	We start by adding the units and write the units digit of the sum and the tens digit of the sum separately.
		5	7				1		
	+	3	9				5	7	
		8				+	3	9	
								6	
Step 2		T	U	Next we add the units and write the unit digit of their sum and the tens digit of the sum separately.	Step 2		T	U	In the 2 nd step we add up all the tens.
		1					1		
		5	7				5	7	
	+	3	9			+	3	9	
		8	6				9	6	
Step 3		T	U	So a 3 rd step requires changing the already written digit in the tens place .	Here we didn't have to change any digit in the answer.				
		1							
		5	7						
	+	3	9						
		9	6						

A similar situation arises in subtraction as well. But because of the variety of cases which arise, I have doubts on whether to teach or even demonstrate this to children.

For division a right to left algorithm is not the most efficient one, as shown below.

Right to left					Left to right				
Step 1		T	U	Let's start with the units, i.e. we divide 6 by 2	Step 1		T	U	We start with the tens, i.e. we divide 3 by 2 and we are left with 1 ten
			3				1		
	2)	3	6				2		
			6						
Step 2		T	U	Now we try to divide the 3 tens . We are left with 1 ten	Step 2		T	U	Now we exchange this ten to 10 units and add the 6 units from the dividend to get 16 units
		1	3				1		
	2)	3	6				3	6	
			6				2		
		3					1	6	
		2							
		1							

Right to left					Left to right				
Step 3			T	U	Step 3			T	U
			1	8				1	8
	2	)	3	6		2	)	3	6
				6				2	
			3					1	6
			2					1	6
			1	0					
 So we have to exchange this remaining ten into 10 units Then we distribute these 10 units  ∴ we have to increase the unit's digit by 5 i.e. change it from 3 to 3 + 5 = 8					 Now we divide these 16 units by 2. Here again we didn't have to change any digit in the answer (or quotient).				

Note: The exchange of units is not obvious in the algorithm and students benefit if the teacher demonstrates the conversion from tens to units by using material manipulatives such as flats and longs or bundles of ten.

So if we show these to students or, even better, get them to work out both ways – left to right and right to left – and then reflect on the efficiency of each method, they will have a much better understanding of why the standard algorithms are the way they are.

I have tried to document how place value is invoked in the standard division algorithm with a detailed step by step demo of both long and short methods simultaneously showing $8643 \div 7$ at <http://teachersofindia.org/en/video/division-pay-attention>.

When the division algorithm is taught, we generally use the shorter version of the division algorithm that violates the subtraction rules that a child has learnt.

Longer version					Shorter version				
Step 1			T	U	Step 1			T	U
			1	0				1	
	2	)	3	6		2	)	3	6
			2	0				2	
Step 2			1	6	Step 2			1	
 Writing the complete 20 doesn't violate the subtraction rule and we are actually subtracting 2 tens or 20*					Writing 20 as just 2 and then subtracting it from 36 is confusing since children may not realize they are subtracting 2 tens from 3 tens				
Step 2			T	U	Step 2			T	U
			1	8				1	8
	2	)	3	6		2	)	3	6
			2	0				2	
			1	6				1	6
			1	6				1	6
 Here we need to change the unit from 0 to 8. However, if this is shown with arrow cards, then this problem can be resolved.					Here we don't need to change the unit. This is why we don't write the unit in step 1.				

Point 3 is important since division is the only place where the standard algorithm requires estimation. For addition, subtraction and multiplication, no matter how large the numbers are, there is no need for estimation.

The strategy is multi-pronged as rounding can go either way. So we need to start with 2 digit divisors and round them to the nearest tens in order to estimate. Naturally, it is a good idea to give children enough practice with divisors that are multiples of 10, e.g. 40, 70 etc., before going into general 2 digit divisors. Next, if the divisor is 62, we can round it to 60 and look for the highest multiples less than the dividend in each step. Similarly, if the divisor is 37, it should be rounded to 40. Of course, numbers ending in 5 like 85 are always tricky – you can use either 80 or 90. It is also important to highlight here that the difference at each step should be less than the divisor. The estimation steps are as follows:

Estimation steps	Example: $256 \div 36$	Example: $256 \div 33$
1. Round off divisor to the nearest multiple of 10	36 rounded off to 40	33 rounded off to 30
2. Estimate quotient (or quotient digit) at that step using the estimate	$256 \div 40$ (or $25 \div 4$) ≈ 6	$256 \div 30$ (or $25 \div 3$) ≈ 8
3. Calculate quotient digit $\times$ actual divisor	$6 \times 36 = 216$	$8 \times 33 = 264$
4. Check a. if quotient digit $\times$ divisor $>$ dividend: decrease quotient digit by 1 and repeat step 3 b. if not, proceed to check dividend – quotient digit $\times$ divisor $>$ divisor: increase quotient digit by 1 and repeat step 3 c. If dividend – quotient digit $\times$ divisor $\leq$ divisor proceed to step 5	Check: a. $216 < 256$ b. $256 - 216 = 40 > 36$ $\Rightarrow$ quotient = $6 + 1 = 7$ Go back to step 3 using 7 as the quotient digit	Check: a. $264 > 256$ $\Rightarrow$ quotient digit = $8 - 1 = 7$ Go back to step 3 using 7 as the quotient digit
5. Complete division step with the (modified) quotient	$7 \times 36 = 252$, i.e. $256 \div 36 = 7$ remainder 4	$7 \times 33 = 231$, i.e. $256 \div 33 = 7$ remainder 25

I welcome suggestions from readers.



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