A Review of

'My search for Ramanujan: How I Learned to Count'

By Ken Ono & Amir D. Aczel

RAMESH SREEKANTAN

Ken Ono is the Asa Griggs Candler Professor of Mathematics at Emory University. Over the last twenty years he has proved many beautiful theorems, many of which explain and generalize the work of Ramanujan which were hinted at in Ramanujan's notebooks a hundred years ago. This book, which is largely an autobiography as told to Amir Aczel, is an attempt on Ken's part to explain, at some level, how it was not only the Mathematics of Ramanujan, but his life's story which influenced Ken and, to a certain extent, made him what he is today.

The book is in three parts, The first part is about his growing up in a suburb of Baltimore, Maryland. Ono's father, Takashi Ono, was a Professor of Mathematics at Johns Hopkins University. He and his wife had moved to the United States in the mid fifties with the intention of being there for a few years and then returning, but at that point the situation for academics in post-war Japan was not so great, so they ended up staying. They had three sons, Momoro, who is an accomplished musician, Santa, who is a bio-chemist and now the president of University of British Columbia, and Ken. Ken was always expected to

follow in his father's footsteps and become a mathematician - something he was not so keen on at that point. This led to some amount of tension between him and his parents - they were what are now called 'Tiger Parents' - Asian American parents pressuring their children to succeed to a fault. This eventually lead to him dropping out of high school at the age of 16 and going to live with his elder brother Santa in Montreal. Around the same time he developed a strong interest in bicycling, and going on long bike rides gave him the peace of mind that he was craving. It helped him escape the pressures he felt his parents put on him. He was also a talented violinist - but as an act of rebellion against his parents he gave it up.

However, before leaving for Montreal, an event took place which turned out to have a profound influence on Ken's life. In early 1984 his father received a letter from India. It was a form letter sent by Janakiammal, the wife of S. Ramanujan, thanking him for his contribution towards the making of a bust of Ramanujan. Takashi Ono was moved to tears by this letter - a fact that surprised Ken as he had not seen his father show such emotion before. His father then told him the remarkable story of Ramanujan's life. This had a great impact on him; and Ken feels that it was this story - of a brilliant Mathematician who had problems with the conventional school and college education - that made his parents allow him to leave school and go spend time with his brother.

The second part of the book is the remarkable story of Ramanujan. Perhaps most readers will be familiar with the fact that Ramanujan was born into a poor family in 1887 in Kumbakonam, now in Tamil Nadu. While he showed exceptional talent in Mathematics, there were no opportunities for a person like him to pursue it. He struggled with the conventional schooling system, though finally, thanks to the patronage of someone who had some understanding of his mathematical genius, he secured the position of a clerk in the Madras Port Trust. While he was there, he wrote letters to several mathematicians in Europe stating his results. All of them ignored him, with the exception of the Cambridge mathematician G.H. Hardy, who, on receiving the letter, thought

that the formulas which Ramanujan had written without proof "must be true as no one would have the imagination to invent them". This led to a correspondence between them, and Hardy then arranged for Ramanujan to come to Cambridge. This resulted in some fruitful collaboration, leading, among other things, to what is known as the *Hardy-Ramanujan Partition Formula and the Circle Method.*

In Cambridge Ramanujan struggled - not only because of the blatant racism that was the norm then - but because he was a strict vegetarian and so did not eat in the dining halls. He cooked his own food, and in those days nutritious vegetarian food was hard to come by. Added to this, the First World War broke out, which led to shortages. His family was far away and apparently his mother did not post the letters his wife had written to him, adding to his isolation. All this took a severe toll on Ramanujan's health; he became ill with what they thought was tuberculosis, though now they suspect it was simply amoebiasis, and this led to his return to India and eventual death, a year later, at the young age of 32.

The third part of the book concerns the last 25 years or so of Ken's life. After spending some time in Montreal, he secured admission to the University of Chicago thanks to the recommendation of a psychologist who was involved in a programme for gifted and talented youth that Ken was in earlier. The University was willing to take a chance on him, notwithstanding that he was a high school dropout. In Chicago he initially took things easy and found the course work hard. It was here that another incident took place which had a singular impact on his later life. A certain visiting faculty member, who had taught Ken *Complex Analysis,* told him that the career of a research Mathematician was not for him; he would be better off being a banker or something else. This upset Ken, as till that point, while he did not care and did not really want to be a Mathematician, the fact that someone said that he was not capable of being one was like a slap in the face. This inspired Ken to take things more seriously and work harder. Around the same time, while flipping channels on TV, he came across a documentary on

Ramanujan, which reminded him of the letter his father had received.

With his new found motivation to prove that professor wrong, Ken did much better in his classes. Paul Sally, a well known professor in Chicago, took interest in Ken's education and helped him secure admission into graduate school at the University of California, Los Angeles. During the time he was in Chicago another important event happened: he met his future wife Erica, who was also a student in Chicago, and they got married shortly before he left for graduate school.

The change in life, from gloomy, cold Chicago to bright, sunny Los Angeles once again made Ken take things easy. He spent time goofing off and cycling till another incident gave him a wake up call. He failed his Algebra qualifier. Though he managed to convince a professor that one of his answers was correct even though it had been marked wrong - and hence he would pass - the professor said that merely passing was not enough for someone who intended to do a Ph.D. in a subject related to Algebra. Once again, this shock was enough to make him take things seriously.

It was around this time that he took an Algebraic Number Theory class with Basil Gordon, whom he managed to impress with a new proof of a known result. Shortly after that he started working with Gordon towards a Ph.D. This was a turning point in his life as the time spent discussing work with Gordon made him discover his love for Mathematics. In a short couple of years he finished his Ph.D. in the area of modular forms - a subject close to Ramanujan's heart.

Towards the end of his Ph.D., a third incident took place, which apparently nobody, not even his wife, was aware of till this book was written. Ken's wife is from Montana and Ken was hoping to move there and get a job in the University of Montana at Missoula. He was invited to a conference close by and took that opportunity to give a talk at the University in the hope that that would help him later on when he applied. Not being aware of his audience, he gave a technical talk, which did not go down well with some of the faculty members. Some old curmudgeon told him that he had wasted his time. This upset Ken greatly; it made him feel that he was a failure who had blown his chances of a job in Missoula. A couple of days later he attempted to end his life by driving onto the lane of an oncoming logging truck. Fortunately he came to his senses and swerved away in the last minute.

Not getting the job in Missoula was a blessing in disguise as shortly after that he received a post doctoral offer from Andrew Granville in Georgia. Granville proved to be an excellent mentor and under his guidance Ken was able to write a very nice paper on Ramanujan graphs. This led to a post-doctoral offer from the University of Illinois at Urbana-Champaign, where he met Bruce Berndt - one of the people in the documentary on Ramanujan, and a person who has spent a large amount of time understanding Ramanujan's *Notebooks.*

It was when he was here that he got an invitation to spend a couple of years at the Institute for Advanced Study (IAS) in Princeton. The IAS is a prestigious institute most famous for the fact that Einstein worked there. It was during this period that Ken's career took off: he wrote a couple of important papers with Kannan Soundararajan and Chris Skinner.

After that, he has not looked back and has done a tremendous amount of work in the areas of elliptic curves and modular forms. More recently he has been working in the area of 'mock modular forms', a field that grew out of Ramanujan's last letter to Hardy, a month before he died. Ken, along with others, has discovered remarkable properties of these forms and they seem to appear in several seemingly disparate areas of Mathematics and even Physics. The book ends with some description of Ramanujan's work as well as some of his own.

As is the case with any autobiography or biography of a successful person - one should always look to see what are the lessons one can take from his or her life. In Ken's case, I think the lesson is how to deal with setbacks. Everyone has them - life never goes as smoothly as one expects - and perhaps what separates exceptional people from ordinary ones

is how one deals with it. Ken used the negative comments of others as inspiration to prove them wrong - and has done so in a spectacular manner. In spite of all these tribulations, he finished his Ph.D. by the age of 24 and was an established researcher, winning a Presidential Early Career grant, within a few years after that. Over the last twenty years he has written over 150 papers! Most mathematicians write about 30 - so this is quite an achievement in itself. He continues to discover remarkable new results in Mathematics - especially related to the Mathematics of Ramanujan.

Another lesson one can take from this book is to be careful with one's words, especially for those who are in a position of authority. The professor who said that Ken was not cut out for research perhaps had not given it a second thought. He perhaps did not realise, till the publication of this book, the impact of his off hand comment on the young undergraduate. In this case it worked out for the best, but that is not always the case. It is often the case for graduate students that their thesis is their whole life, and so feeling that they have failed in some way can have disastrous consequences.

This book, which is very open and honest, does give you the example of someone who struggled initially but was able to overcome those difficulties to succeed spectacularly. I think it would be beneficial for an undergraduate or early graduate student who is plagued with doubts as to whether he or she is 'good enough' to pursue the path they have chosen, to read this book.

In my personal opinion Ken was perhaps an exceptional person all along - his father perhaps recognised his mathematical talent before he himself did - and much of his fear of failure was caused by self-doubt rather than reality. Then again, at the end of the day, it is only one's perception of oneself that matters and being able to control this is perhaps the secret of success.

Another point about Ken's life which is perhaps relevant in the Indian context is the importance of physical exercise. Many Indian students, especially in their late teenage years, give up any sort of physical activity because they think they have to study and have no time for it. In fact, studies show that exercising regularly is very good for the brain, and Ken - who is almost a professional triathlete at the age of 48, is living proof of it.

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A word about partitions

Much of the work of Ramanujan and Ken Ono is perhaps too difficult to describe. However, some of their work centres around the study of **partitions of numbers.** The partition of a natural number n is an expression of n as a sum of non-decreasing sequence of natural numbers. The function $p(n)$ denotes the number of partitions of n.

For example:

$$5 = 1 + 1 + 1 + 1 + 1$$
$$= 1 + 1 + 1 + 2$$
$$= 1 + 2 + 2$$
$$= 1 + 1 + 3$$
$$= 1 + 4$$
$$= 2 + 3$$
$$= 5$$

So $p(5) = 7$. While the first few values are easy to compute, the number $p(n)$ grows rapidly. For example $p(100) = 190, 569, 292$. A natural question, then, is whether there is a formula for $p(n)$. That may be too much to expect, but Hardy and Ramanujan discovered a remarkable asymptotic formula for $p(n)$. Ken Ono and his collaborators discovered another arithmetic formula for $p(n)$.

Another set of results which Ramanujan had arrived at concerned congruences satisfied by $p(n)$. He proved, for instance, that for any non-negative integer k, $p(5k + 4) \cong 0$ mod 5; i.e., $p(5k + 4)$ is always divisible by 5. He also discovered that there were similar properties for the primes 7 and 11, but also made the remark that there were no 'easy' congruences for higher primes - namely of the form $p(mk + r) \cong 0$ mod m. In the 1960s Atkin discovered that $p(11^3 \cdot 13 \cdot k + 237) \cong 0$ mod 13. One of the remarkable results of Ono concerned showing that there exist congruences modulo every prime. Further, along with Scott Algren, he showed that there exist congruences modulo every integer co-prime to 6.