

The breaking of the IM CODE

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This is a fictional account of how the city of Mumbai was spared one more trauma of serial bombings one hot April afternoon.

IPS Officer and Joint Commissioner of Police K. D. Abhyankar was in charge of the Anti-Terror Squad in Mumbai Police that summer. He had just finished his afternoon cup of tea at 3 pm when his cellphone gave out its usual ring tone singing *Hum Honge Kamiyaab* (*English: We Shall Overcome*). The phone call informed him of the discovery of a suspicious package in a dustbin in the lobby of the sprawling Maharashtra Assembly Building (Maharashtra Vidhan Bhavan) at Nariman Point in the city.

Mr. Abhyankar immediately swung into action. He sent a bomb disposal squad and an ambulance to the spot, called his colleague, Joint Commissioner of Police (Law & Order) to dispatch enough policemen to keep people out of harm's way and to alert all police stations in the city and the Railway Police to mount a minute search in their areas for any suspicious packages. He also alerted all the hospitals in the Fort area and arranged to alert all the other hospitals, ambulance services and fire stations in the city to prepare for any emergency that could arise. Then he called for his car to proceed to the Assembly Building.

As Mr. Abhyankar was about to leave, his secretary, Miss Bhadkamkar came running out shouting, "Sir, there is an important e-mail message that might be of crucial importance." Her boss grabbed the print-out of the message from her hands, barked that she knew what to do and who to get to work on tracing the e-mail message - "So get on with it pronto!" He himself got on to what he wanted to do, namely reach Ground Zero.

Keywords: code, clue, grid, ordered pair, coordinate geometry, angle

Abhyankar's office was in the old Commissionerate Building near Byculla in Mumbai and it was a 25 minute drive to the Assembly Building. But in those 25 minutes he carefully went through the brief but explosive e-mail message. The entire text of the message (in near-perfect English) was the following:

Mr. Abhyankar, we have planned something bhayankar for you today. Your beloved city would erupt from various spots later this afternoon. Which spots? You can try your best but aap dhoondhte reh jaoge! What you would find is not what you would be looking for but body parts of hundreds of people. How terrible that people would hate you and pounce on you for not preventing this tragedy, poor you! We are really sorry and feel like helping you. OK, here is some hint. See if the following means anything to you.

*(7, A, 6), (9, B, 6.15), (8, C, 5.30),
(12, E, 6.30), (8, F, 5.20), (15, K, 6)*

Ok, get to work, Cop, and earn your pay!

The message was signed "IM Avengers."

Mr. Abhyankar experienced cold and silent fury as he finished reading the message. It was common for terrorists to send e-mail messages to media just as bombs were blasting off but this was unusual. This message was sent well before the blasts and it was sent to the police. The messages to the media never mentioned the number or the locations of the bombs but this one had the gall to do even that. Apparently the terrorists were emboldened by their frequent successes and were in the mood to jeer at and challenge the authorities to thwart their plans. Abhyankar decided that enough was enough and that this challenge would be met. He then turned his attention to the list given in the message.

*(7, A, 6), (9, B, 6.15), (8, C, 5.30),
(12, E, 6.30), (8, F, 5.20), (15, K, 6)*

He saw that apparently the explosive devices had been placed at 6 locations. The last number in each triple obviously indicated the time of exploding the device. The 6 blasts were scheduled between 5:20 pm and 6:30 pm. Recalling his

school geometry and geography, Abhyankar thought for a second that the first two entries in each triple probably fixed the exact location of each device, perhaps something like the longitude and latitude of the site. Just as he thought this, he arrived at the site where the suspicious package had been found.

The police had evacuated all the people from the vicinity. It was 3:25 pm and the bomb squad was busy defusing the bomb. In 5 minutes, the bomb was defused. Mr. Abhyankar anxiously asked as to when the blast was set to go off. "Sir, at precisely 5:20." That was the first break Abhyankar got. So this was the penultimate triple (8, F, 5.20) on the list.

It appeared that his hunch that the last entry in every triple was the time co-ordinate had been right. This also meant that the next blast was due at 5:30, just 2 hours away, and then the other blasts were due in quick succession. So the problem was to locate all the remaining 5 sites in about 90 minutes, send bomb disposal squads to each location and defuse the devices before they went off.

Now what really were the first two co-ordinates? Of course they were space co-ordinates but they could not be latitude and longitude because both would then be numbers. Here, the second was a letter of the English alphabet and, in any case, for locations within a single city, the latitude and longitude wouldn't differ by very much. Therefore, they would be useless to distinguish between two locations in the same city. Abhyankar did not know in what other way the locations of places on the earth could be indicated. But he had the presence of mind to remember that most tourist maps of cities identified important landmarks, the airport and hotels in the city by a pair such as D-5 or (5, D). Such maps of a city always divide the city into a grid of squares. The horizontal squares are numbered 1, 2, 3, etc., and the vertical squares are labelled A, B, C, etc. (or the other way around.) On a tourist map of Mumbai, for instance, Mumbai Central Station could be H-3 or (3, H), and so on. Just then City Police Commissioner Tavade arrived. Abhyankar

apprised him of the e-mail message and his thinking on it. Tavade said, “OK, what are we waiting for? Let us get hold of a tourist map of the city.” Fortunately, the Assembly Library had such a map. The map had been divided into ten horizontal rows of squares 1, 2, 3, etc., and eleven vertical columns of squares which were labelled A, B, C... K. But there were problems. The bombsites (15, K) and (12, E) did not exist on the map and the bombsite (8, F) was in the Arabian Sea off the Sewree coast! So at least the terrorists had not used this particular grid reference system. Tavade further asserted that no matter what map they had used, the spot (15, K) was way down South and to the far left or far right. That meant it must be in the extreme lower left or extreme lower right corner of the map. These two corners in any map of Mumbai had to be in the sea as South Mumbai was a very narrow piece of land. Both the police officers came to the conclusion that the number and alphabet listing were red herrings designed by the terrorists to put the officers on the wrong path. In fact, the list of the bombsites was in some sort of code and they had to seek the help of a mathematician to break the code. Without wasting any more time, Tavade who happened to be a Mathematics graduate took Abhyankar to the Mathematics Department of the nearby Institute of Science. They met Professor Pandit there. Pandit took just a few minutes to understand their problem. He told the officers that in all probability the terrorists had used the simplest substitution code – they had substituted letters for numbers – A for 1, B for 2, C for 3, and so on. Assuming that this is true, we have the following problem:

The bombs are planted at points whose co-ordinates are (7, 1), (9, 2), (8, 3), (12, 5), (15, 11) and (8, 6). We know where the last point is. It is the State Assembly building. The problem is to find where the other points are. The important thing is to realize that we do not know where the origin is and what the directions of the axes of X and Y are.

Professor Pandit’s reasoning took flight. He said, “If we can only locate the origin and the two axes, we can then locate any given point. Given a point

(a, b), how do you find the origin?” Even in those tension filled minutes, Tavade smiled a little as he noticed that like a typical Mathematics Professor, Pandit was more at home with a general point (a, b) than the point (8, 6)!

Now Professor Pandit was in his element. Immersed in his problem and quite unmindful of his two visitors, he was carrying on a dialogue with himself. He muttered, “At least I know that the distance of the origin from the point P whose co-ordinates are (a, b) is $\sqrt{a^2 + b^2}$. If this number is c , then the origin is at a distance c from P. So the origin O is on a circle with centre P and radius c .” Explaining this to his visitors, he told them, “You see the origin is at a distance 10 from the point (8, 6) because $8^2 + 6^2 = 10^2$.” This was Greek and Latin to Abhyankar who had graduated with History and Political Science, but he went along with Tavade in nodding his head. Pandit continued, “So the origin is somewhere on the circle with centre (8, 6) and radius 10. Of course that complicates matters because it means that there are infinitely many possibilities for the origin.” His visitors looked at him open-mouthed, not comprehending his last assertion.

The bewilderment of the policemen meant nothing to the mathematician. He argued with himself, “All I know is that the origin O is some point on the circle with centre P (8, 6) and radius 10. But it is possible that not all points on the circle can qualify to be the origin. Hopefully only a few points on the circle and perhaps miraculously, only one point on the circle may

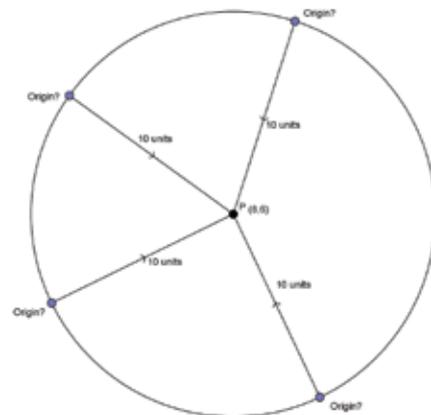


Figure 1: The origin could be at any point at a distance of 10 units from P

qualify as the origin. In that case, we would be nearly home!”

He went to the black board, took a point on it, labelled it $P(8, 6)$ and drew a circle with P as centre and radius 10, labelling the circle ω . “Now let us just take any point on this circle ω , call it O and let us see if we can choose lines OX and a perpendicular line OY so that for these axes, the point P would really have the co-ordinates $(8, 6)$. Now the y -coordinate of P is 6, so P is at the distance 6 from OX and likewise it is at the distance 8 from OY .” He kept quiet for a few seconds and the policemen looked at their watches. It was 4:15 pm and time was running out on them. They just prayed that the mathematician would deliver in time.

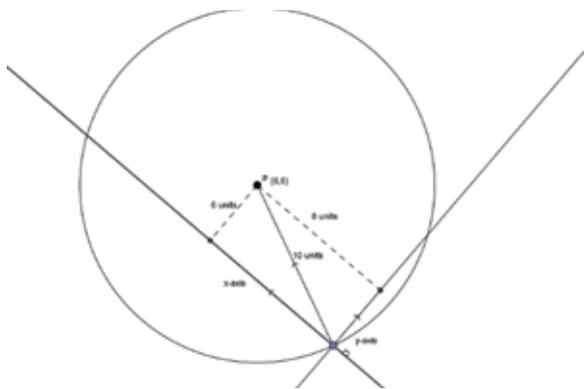


Figure 2: If O is the origin.....

“Oh yes,” Pandit resumed after a few seconds, “it means that if I take two other circles, say μ and β with the same centre P and radii 8 and 6 respectively, then I can take the tangent to the circle β which passes through the external point O and take that as my X -axis because it would be at the distance 6 from P . Likewise, I can take the tangent to the circle μ which passes through O and take that as my Y -axis. Now what about the angle between these two tangents? If that is a right angle, then I can choose these as my axes. Is it not?” Again Pandit became silent for a while. The two visitors fretted and fumed and bit their nails but their host was oblivious to their moods. Just then, Tavade got a call from a TV News Channel wanting him to confirm that a terrorist attack on the city was imminent. “I have no such announcement to make,” said Tavade. The reporter was persistent, “Do you deny that your

men are scouring the city trying to find bombs?” “Well, good for them. They have been told to be always on the lookout for suspicious objects,” said Tavade and ended the call. This was a new complication. The media hounds would now be after them all the time. Tavade asked the Police Control Room to divert all his important calls to Abhyankar’s number and switched off his own cell phone.

Pandit chose the two tangents on the opposite sides of the line OP , called the points of contact as M and N and immediately saw that triangles OMP and ONP were congruent, both having sides 8, 6 and 10. From this, he saw in a flash that angle MON is indeed a right angle. Pandit’s face fell for the first time as he said, “I am afraid, every point on the circle ω is a potential origin and the situation is hopeless. There are infinitely many choices for the locations of each of the other points.”

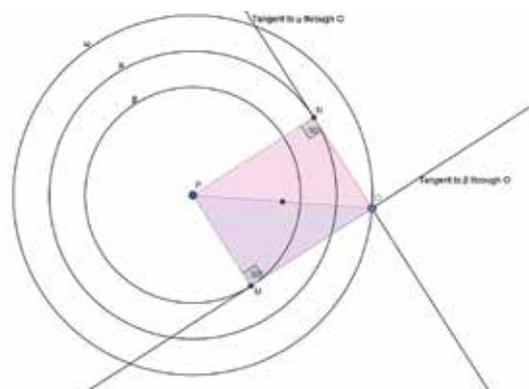


Figure 3: Constructing tangents to the two circles through the external point O

Abhyankar and Tavade had been worried since the beginning but now they were in a panic. If the home minister and the CM (not to mention the media) came to know that their two top police officers had wasted precious time in the company of a mathematician while **five** bombs were merrily ticking away somewhere in the city, they would be furious. Perhaps the ministers would have preferred them to spend time with an astrologer!

But Pandit was still thinking. He told his visitors, “You have given me the location of only one point. If you can give me the location of one

more point, call it Q, then I know the distance of two of the points from the origin. I can therefore construct two circles centred at P and Q. I can restrict the origin to lie on these two circles and as two circles intersect in at most two points, I can tell you the locations of the remaining 4 points with just some uncertainty. And if you can give me the location of a third point also, then I can exactly locate the origin and the problem would be easily solved." But there was no news of any more discoveries of bombs from the city and so the policemen were helpless. There was nothing to do but to wait and hope that soon the bands of policemen who were combing the whole city would come up with some "good"(!) news.

It was 4:30 pm and the clock was ticking on relentlessly. Now they had barely half an hour to crack the code. The three men were under great tension. The mathematician was passing time and pretending not to be tense by doing some silly calculations and writing down some numbers on a piece of paper. Tavade kept busy on his phone giving instructions to mobilize a large number of police personnel with trained dogs at various spots in the city to search out a bombsite at a moment's notice. That left Abhyankar to bite his nails and to dread a possible bleak future for himself. But he too called his officers and asked for a judicious locating of the many bomb disposal squads so that one of them could quickly reach any given site.

The ringtone *Hum Honge Kamiyab* sounded at 4:36 pm. Abhyankar was informed that a suspicious looking parcel was found among the vendors of bhelpuri at Chowpatty sea beach. The bomb squad waiting at Royal Opera house was sent there and Abhyankar turned to Pandit with some hope. But Pandit told him, "We still do not know the co-ordinates of this new site. At least wait until we know when the bomb there was set to go off."

The bomb at Chowpatty was deactivated at 4:55 and it was learnt that it was to explode at 6 pm. Tavade exclaimed, "Oh my God, there are two locations on the list that were scheduled to explode at 6, so which one is this? Is it (7, 1) or (15, 11)?" Pandit's random and nervous calculations now came into play. He said, "It

can be neither. Because according to the formula for the distance between two points, the distance of (7, 1) from the known location (8, 6) is $\sqrt{26}$ (approx. 5) and that of (15, 11) from the same point is $\sqrt{74}$ (approx. 8.6) - much more than the actual distance between Assembly building and Chowpatty (about 4 km by road)." This was another blow. Abhyankar was stunned because to him this indicated that there were bombs at sites that were not on his list. It was obvious to him now that there was no way he could avoid the catastrophe.

But Pandit thought like a mathematician. He said, "We have assumed all along that the terrorists have used the Cartesian co-ordinate system to identify points. This is the system in which there is an origin and two perpendicular lines called axes through it. Co-ordinates of any point in the plane are the distances of that point from these two axes. It is entirely possible that they may have used the polar co-ordinate system. In this system there is a definite point O called pole and a ray OX passing through O. The polar co-ordinates of any point P (other than O) are (r, θ) where r is the distance OP and θ is the angle XOP. So in this system the distance from the pole is the first number and the second number is an angle."

Pandit looked at the list of points again and noticed that the second numbers were all between 1 and 6 with the sole exception of 11. It immediately struck him that these need not be angles. Could they be numbers on a clock face?

The second number is not the angle XOP, but if O is the centre of an imaginary clock dial on the ground, the second number is the number on the dial indicated by the line OP where P is a bombsite. As none of them is between 6 and 11, it was clear that most locations are to the East of the "clock," only (15, 11) being slightly West of North. This would indicate that the pole (and also the imaginary clock) was almost on the West Coast of the city. Assuming then that the second numbers were really numbers on a clock face, we get that, as the Assembly building is (8, 6), it is exactly below (to the South of) the "clock" and (15, 11) is almost due North.

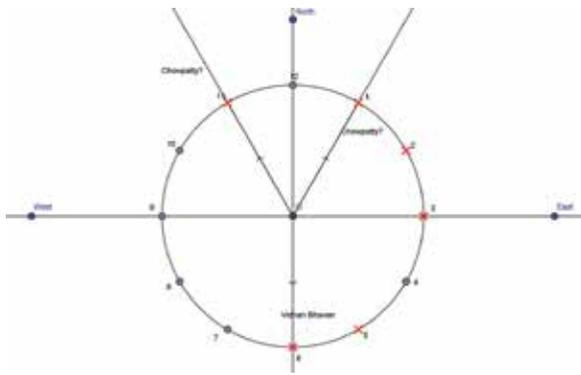


Figure 4: The clock

So he said, “I have a hunch that the clock face is about one-third of the way from the Assembly Building to Chowpatty. Let us see a map of the city.” No map was available in Pandit’s office, so he opened the Google map on the internet. As he zoomed in to the city and looked between Nariman Point and Chowpatty, he and the two police officers jumped with joy on seeing a clock face stare up to them in the face. It was Wankhede Stadium, almost a perfect circle. They had no doubt whatever that all the first numbers were distances from Wankhede and the second number indicated the direction on the clock face. It took them another 5 minutes to realize that the first numbers were not the actual distances but they were scaled up (or down depending on the units you use) by some constant factor.

Once this was understood, it was a matter of a few minutes to locate all the remaining sites. The time was already 5:15 pm and one of the bombs was due to blast off in 15 minutes and it was determined to be somewhere in the Chatrapati Shivaji Railway Terminus.



Figure 5: The map!

There was not enough time to find the exact site and then to defuse the bomb. But Tavade’s large contingents of policemen and trained dogs were standing by to find the bomb and Abhyankar sent, not the usual BDS (Bomb Disposal Squad), but the new TCV (Total Containment Vehicle) to the site. A Robot picked up the bomb, put it gently in a hollow but strong iron sphere on the TCV and locked the sphere securely just before the blast. At precisely 5:30 the bomb blasted off but the blast was completely contained within the sphere and there was no damage to the containing sphere or to the TCV or to the surroundings. The BDS’s were quickly sent to the other sites to get the bombs deactivated. The last bomb was defused at 5:50 pm and the police officers finally sighed with relief. At 6:15 pm, the CM called Abhyankar (again the *Hum Honge Kamiyab* ring tone) to congratulate him on his brilliant success.

The next day, Abhyankar found the time to call up and thank Professor Pandit for his help. Mathematician that he was, Pandit replied, “Oh, that? That was fine but you know, actually the terrorists had used polar co-ordinates and they had scaled the distance co-ordinate as we had found, but I later saw that they had even scaled down the angle co-ordinate by a factor of 30.” Abhyankar realized that he had another reason to be thankful for and that was that he did not have to listen to Pandit’s mathematical gobbledygook anymore! However he was game enough to admit to himself that but for Pandit’s mathematical skills, he would be in the dumps right then. “So let Pandit be, and may his tribe increase!” he thought.

Mumbai escaped a horrible fate by the skin of its teeth but Mumbaiers did not even know it because the story was kept under wraps until the terrorists who planned this event were captured after many more blasts in many cities and after many months.



ARUN VAIDYA retired from Gujarat University, Ahmedabad as Professor of Mathematics. However, he continues to live an active mathematical life. He worked on problems of Number Theory and Mathematical Modelling. He also edited the journal *Mathematics Student* and the Gujarati mathematical periodical *Suganitam*. He has been deeply involved in the search for Mathematical Talent at all levels. He was the main organizer of the International Mathematical Olympiad hosted by India in Mumbai in 1996.

The Code :

(7, A, 6), (9, B, 6.15), (8, C, 5.30), (12, E, 6.30), (8, F, 5.20), (15, K, 6)

The Sequence of Reasoning:

The last number represented the time at which the bomb was scheduled to go off. This was validated when the first bomb was defused and found to have been set to detonate at 5.20 p.m.

The first two numbers were space coordinates indicating the sites at which the bombs had been planted.

They could not be the latitude and longitude as these cannot vary so much in a single city.

They could be the grid references on a tourist map. However, this meant that (15,K) would be in the sea as Mumbai was a narrow strip of land.

Professor Pandit suggested that the letters corresponded to numerals, i.e., A→1, B→2 and so on.

In that case, these could represent the Cartesian coordinates of the bombsites. If the origin was known, then each site could be located.

So the first step of the decoding seemed to have been cracked!

(7, 1, 6), (9, 2, 6.15), (8, 3, 5.30), (12, 5, 6.30), (8, 6, 5.20), (15, 11, 6)

But only one point, i.e., (8,6) was known.

The origin could be anywhere on a circle ω centered at P (8,6) with radius 10 (by Pythagoras' theorem).

Drawing tangents through a point O, to two circles centered at P with radii 8 and 6 respectively, makes angle MON a right angle.

(M and N are the points of contact of the tangents and the circles). If O is the origin, then the two tangents could represent the x and y axis as point P is at a perpendicular distance of

8 units from one of them and 6 units from the other. But O could be any point on ω !

Knowing the location of one more bomb site would restrict the origin to one of two points since the origin would now have to lie on two circles of known radius and known centres. And knowing one more bomb site would fix the origin since three circles can have only a single point of intersection.

When the second bomb site was identified to be one of (15,11) or (7,1) they realized that the distance of (8,6) from either of these points was greater than the actual distance between the sites.

At this point, Professor Pandit suggested that a polar coordinate system could have been used instead of the Cartesian coordinate system.

On noticing that the second 'coordinate' was between 1 and 12 for all 6 points, Professor Pandit thought that these numbers could represent numbers on a clock face.

Since Vidhan Bhavan was at 6'o' clock and Chowpatty at either 11'o' clock or 1'o' clock, the clock had to be situated between these, and since the distance from Vidhan Bhavan to the clock was 8 units and from Chowpatty to the clock was either 15 units or 7 units, Professor Pandit acted on his hunch that the clock face is about one-third of the way from the Assembly Building to Chowpatty and found a clock at Wankhede Stadium.

This helped them crack the puzzle and locate the remaining bombs!