

SHORT PROOF OF AN INEQUALITY FOR THE AREA OF A QUADRILATERAL

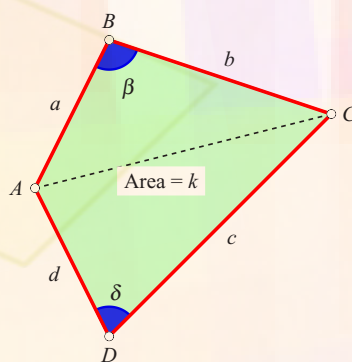
MOSHE STUPEL & DAVID BEN-CHAIM

"SHAANAN" – Academic Religious Teachers College, Haifa.

Here is an elegant and remarkably compact one-line proof for an inequality relating to the area of a quadrilateral:

Theorem. *The area of a quadrilateral is less than or equal to a quarter of the sum of the squares of the sides. Equality holds precisely when the quadrilateral is a square.*

Proof. With reference to the figure shown below, with sides, angles and area as indicated:



$$k = \frac{ab \sin \beta + cd \sin \delta}{2} \leq \frac{ab + cd}{2} \leq \frac{\frac{a^2 + b^2}{2} + \frac{c^2 + d^2}{2}}{2} = \frac{a^2 + b^2 + c^2 + d^2}{4}.$$

Equality holds precisely when $\beta = 90^\circ = \delta$ and $a = b, c = d$.

□

Keywords: Quadrilateral, area, inequality