

# HAPPY NUMBERS

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In this short note, we talk about *Happy Numbers*. As this has to do with the digital representation of a number, we must specify right at the start which number base we are working in. So please note: *We will be working throughout in base ten.*

**The iteration.** Start with any positive integer and compute the sum of the squares of its digits. Do the same for the number that results from this operation; then repeat for the new number, and continue this iteration. (Note from the editor: An ‘iteration’ is simply some operation or some step that you do over and over again. Mathematicians have studied many different kinds of iterations. A particularly well-known iteration is the one used in Euclid’s GCD algorithm.) Let us call this the *SSQ iteration* (SSQ for ‘sum of squares’).

For example, suppose we start with the number 2375. The sum of the squares of its digits is  $2^2 + 3^2 + 7^2 + 5^2 = 87$ . The sum of the squares of the new number is  $8^2 + 7^2 = 113$ . Continuing, here is what we get:

$$\begin{aligned} 2375 &\rightarrow 87 \rightarrow 113 \rightarrow 11 \rightarrow 2 \rightarrow 4 \rightarrow 16 \rightarrow 37 \rightarrow 58 \\ &\rightarrow 89 \rightarrow 145 \rightarrow 42 \rightarrow 20 \rightarrow 4 \rightarrow 16 \rightarrow 37 \rightarrow 58 \\ &\rightarrow 89 \rightarrow 145 \rightarrow 42 \rightarrow 20 \rightarrow 4 \rightarrow 16 \rightarrow 37 \rightarrow 58 \\ &\rightarrow \dots \rightarrow \dots \end{aligned}$$

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Observe that we have got caught in a cycle: the sequence of numbers 89, 145, 42, 20, 4, 16, 37, 58 recurs indefinitely. (You will see why we call it a **cycle**; it is a closed loop.)

Let us now start with some other number, say 55. Here is the sequence of numbers which it yields:

55, 50, 25, 29, 85, 89, 145, 42, 20, 4, 16, 37, 58, . . . .

We are caught in the very same cycle again (89, 145, 42, 20, 4, 16, 37, 58, . . . )!

Experimentation with other starting numbers shows that we end up in this cycle very often. However, it is not the only possible outcome. For

example, let us start with the number 49. Here is what it yields:

49, 97, 130, 10, 1, 1, 1, 1, 1, 1, 1, 1, . . . .

Note what has happened: at some point, the SSQ value turns out to be 1, and once this happens, we will never budge from 1; for:  $1 \rightarrow 1 \rightarrow 1 \rightarrow \dots$

Here is another example which yields the same outcome. Let us start with the number 193; here is what it yields:

193, 91, 82, 68, 100, 1, 1, 1, 1, 1, 1, 1, . . . .

At this point, we ask: *For which starting numbers does it happen that we ultimately get stuck at 1?*

Interval	Happy numbers in this interval
1–100	1, 7, 10, 13, 19, 23, 28, 31, 32, 44, 49, 68, 70, 79, 82, 86, 91, 94, 97, 100
101–200	103, 109, 129, 130, 133, 139, 167, 176, 188, 190, 192, 193
201–300	203, 208, 219, 226, 230, 236, 239, 262, 263, 280, 291, 293
301–400	301, 302, 310, 313, 319, 320, 326, 329, 331, 338, 356, 362, 365, 367, 368, 376, 379, 383, 386, 391, 392, 397
401–500	404, 409, 440, 446, 464, 469, 478, 487, 490, 496
501–600	536, 556, 563, 565, 566
601–700	608, 617, 622, 623, 632, 635, 637, 638, 644, 649, 653, 655, 656, 665, 671, 673, 680, 683, 694, 700
701–800	709, 716, 736, 739, 748, 761, 763, 784, 790, 793
801–900	802, 806, 818, 820, 833, 836, 847, 860, 863, 874, 881, 888, 899
901–1000	901, 904, 907, 910, 912, 913, 921, 923, 931, 932, 937, 940, 946, 964, 970, 973, 989, 998, 1000

Table 1. List of happy numbers from 1 to 1000

Numbers with this property — that the SSQ iteration starting with that number ultimately reaches 1 and gets ‘stuck’ there — have been called *Happy Numbers*.

Is there a formula to generate happy numbers? There does not seem to be any formula. Table 1 lists all the happy numbers that do not exceed 1000. We generated the list manually!

Note the large number of happy numbers between 301 and 400, between 601 and 700, and between 901 and 1000. Note also the small number of happy numbers between 501 and 600. The wide variation is very puzzling!



**DEV VINOD CHANDAN** is currently studying in Grade 10 at SAS Billabong High School, Mahim, Mumbai. He worked on Happy Numbers as part of Vinay Nair’s “Vedic Maths Camp” at Chinmaya Ashram, Pune in May 2016. He lives with his parents and younger brother in Mumbai. He is an avid maths and science enthusiast. He enjoys playing cricket, table tennis and football. He is an eager yoga student and also learns the keyboard. Dev may be contacted at [devvchand@gmail.com](mailto:devvchand@gmail.com).

**Note from the editor:**

Adhiraj Patil is from SAS Billabong High School, Mahim, Mumbai. We have been unable to get his photograph and key biographical information; hence this brief write-up.

The above article was written as a result of a Vedic Maths workshop done by Vinay Nair at SAS Billabong High School. Dev Chandan and Adhiraj Patil were assigned the task of exploring the notion of Happy Numbers. The project was to be completed within 2-3 days. Further, the investigators were to use manual calculations only; no use was allowed of either calculators or cell phones or any kind of computer devices. So they got down to their task by dividing the numbers between them and then physically checking each and every number and cross-checking the same. This article has arisen as a result.