Azim Premji University

A publication of Azim Premji University
together with Community Mathematics Centre,
Rishi Valley

TEACHING GEOMETRY-II

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AT THE PRIMARY LEVEL

At Right Angles
A Resource for School Mathematics

[...Continued from the previous issue]

In many ways, the teaching of geometry when approached in the right way holds an immense potential for learning the art of seeing and observing. As one begins to play around either with a plain square paper by folding it in different ways or connecting dots in a dot paper one begins to see a variety of shapes emerge. Personally I have always found it to be a pleasurable and enriching experience as every student that I meet sees and notices shapes in his or her own special way. There seems to be a lot of scope for developing an eye for seeing, for improving observational skills and for bringing out one's creativity. Also amongst all the topics in mathematics it is in the teaching of geometry that a teacher can use a hands on approach to a high degree to discover properties of shapes, the manner in which they behave when they are moved around, rotated, slid, the way they can be dissected and reassembled to form new shapes, etc. In fact, once the teacher introduces an activity within no time children will develop their own variations of it and begin to make their own explorations. There is no greater reward that a teacher can obtain than that.

CLASS TWO

As children move into higher grades one may continue to use the same materials used in the earlier years (shapes, geo-boards, straws, dot paper, paper plates, tangram sets) but one raises the level of challenge in each activity.

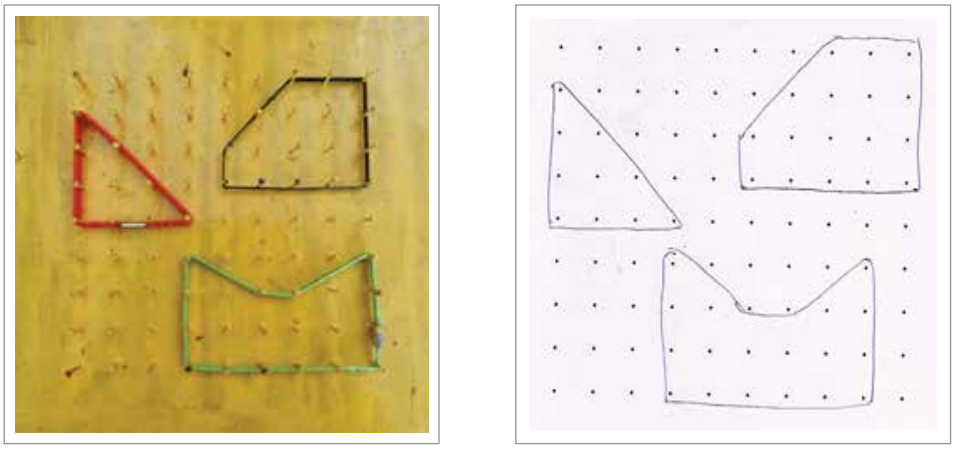


Figure 1

Geo-board

For instance: in activities involving usage of a geo-board, the teacher can require the children to record the shape they have created on a geo-board on a dot paper. Simple as it seems, many children find it a complex task. They need to visualise the corresponding points of their shape on the geo-board on the dot paper. They need to have a sense of the size of the shape and its orientation; the number of pegs enclosed vertically and the number of pegs enclosed horizontally; the straightness of the sides, etc. This leads up to the idea of mapping which comes much later.

PATTERN CREATIONS ON DOT PAPER

Teacher can now follow up the geoboard activities with more dot paper activities.

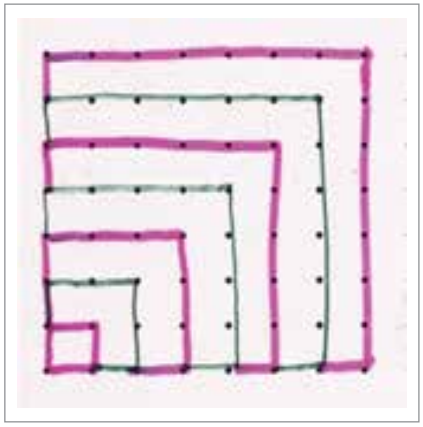


Figure 2

They can use square dot paper to create shape patterns. They can draw series of same sized squares in different colours, alternating series of squares and rectangles or alternating series of squares and triangles, series of squares of increasing size (concentric or starting from the same corner), and so on (Figure 2).

They can try more complex patterns involving squares bordered with triangles on all sides.

They can make patterns which result in a circular or a closed formation, and they can also make patterns which continue to expand.

In the process of building these patterns they use their intuitive understanding of symmetry, similarity, tessellations etc.

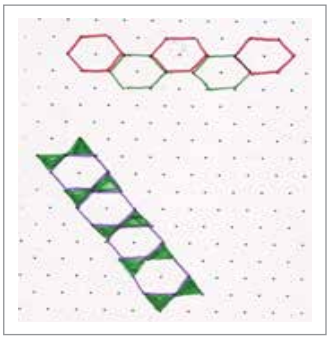


Figure 3

In a similar manner, triangular dot paper can be used to create many patterns using triangles and hexagons (Figure 3).

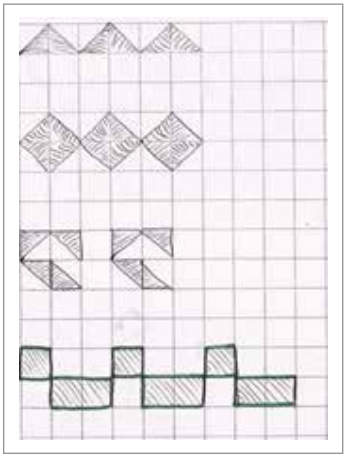
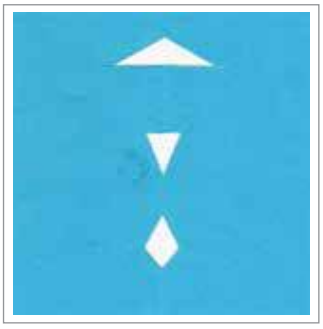


Figure 4

The teacher can give some patterns of shaded squares (Figure 4) and ask the children to copy them. This too requires them to observe small areas carefully and builds spatial abilities.

PAPER FOLDING AND SYMMETRY



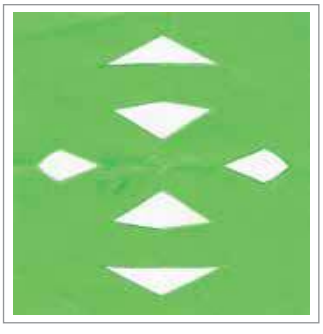


Figure 5

Let them use paper folding and cutting to create symmetrical shapes (Figure 5).

Initially they can explore cutting papers made with one fold and see the shapes that emerge. Later they can make two folds, vertical and horizontal, and observe the shapes thus created.

Through experimentation let them discover the different types of cuts and the shapes that result due to these cuts.

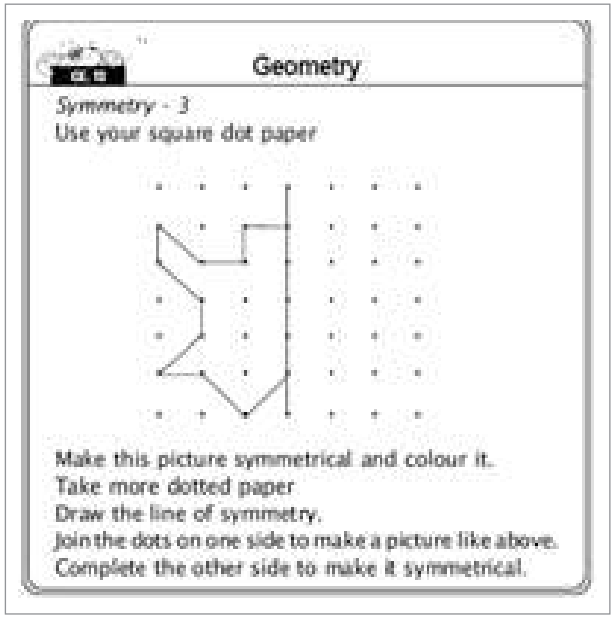


Figure 6

Continue to explore symmetry by making half-complete designs. Let children copy them onto dot paper and complete the other side (Figure 6).

Various simple rangoli patterns can be taught and created.

They can do further work on symmetry through ink designs or paint designs on folded paper.

TANGRAMS

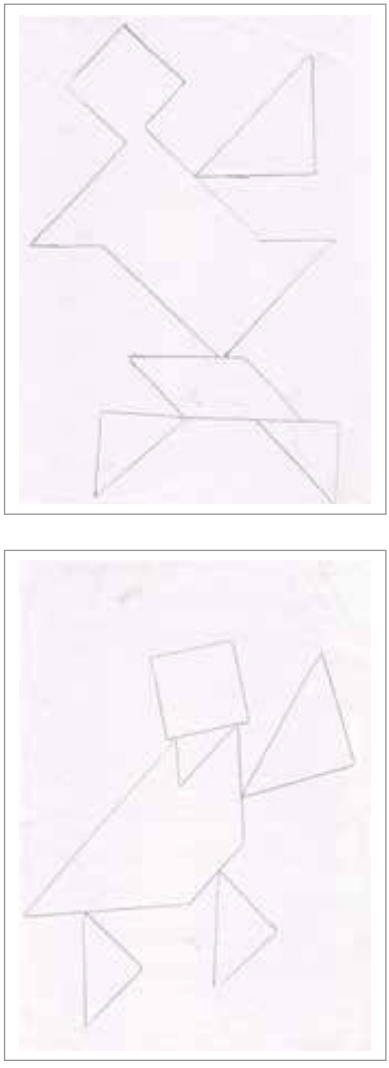


Figure 7

Show them some designs (select simple ones as shown in Figure 7) and require them to build these designs. Initially give designs with the outlines drawn to size on a large chart paper so that children can fit the tangram pieces onto the outline. Children often solve them using intuition. In the process of deducing the pieces to be used to build the given design, they will need to observe the design carefully: the corners, slant of the lines, size of the shape, etc.

CLASS THREE

3-D STRUCTURES

How similar? How different?

Discuss the 3-D structures they see around in the class room. Some shapes are regular 3-D shapes: cuboid, sphere, etc. But some shapes we use resemble cuboids or cylinders closely but are not exactly cuboids or cylinders. *Example:* A water bottle is often not a complete cylinder but narrows down at the opening. Discuss: "In what way is it different from a cylinder?" A tiffin box may look almost like a cuboid but may not be exactly a cuboid. Similarly, a compass box is generally curved at the corners. We need to recognise that while doing this activity children are closely observing, comparing and contrasting both the similarities and the differences.

Building skeletal models of 3-D shapes

Normal sturdy drinking straws or paper straws can be used to build skeletal models (Figure 9). Children can be taught to roll small paper sheets tight and make them into paper straws. They make excellent material for various activities. Children can build cubes, cuboids, prisms, pyramids etc. with them by joining them with small rubber tube connector pieces.

Ask: "How are these different from solid cubes or cuboids?" Introduce simple vocabulary like faces, edges, corners, straight lines, slant lines etc. (See Figure 10.)

Ask: "Where do these two faces meet?" "Where do these three faces meet?" "Can you show me two faces which do not meet?" "Which face is opposite to this face?" "Which solid shape has most edges?" "Which solid shape has edges of the same length?" "Which solid shape has edges of different lengths?" "Which solid shape has only square faces?" "Which solid shape has rectangular faces?"

Ask questions about where two edges meet. For example in the case of a tetrahedron ask: "How many V joints do you see?" "How many L joints do you see?" "Do you see any other kind of joints?"

Let them contrast shapes like cuboids and cubes with shapes like prisms and pyramids (naming is unnecessary).

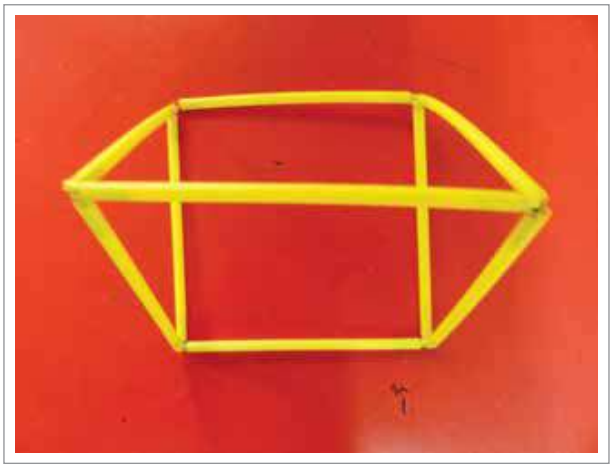


Figure 9

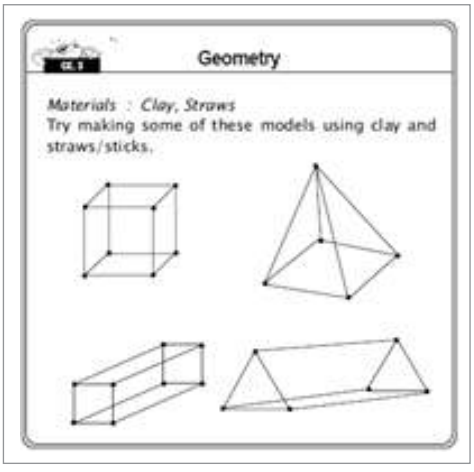


Figure 10

Building 3-D shapes from 2-D shapes

Let them stack up many wooden triangle pieces and see the solid shapes they can form (e.g., a prism).

Let them stack up many wooden rectangle pieces and see what shapes they can form (e.g., a cuboid).

Let them stack up many square wooden pieces and see the solid shapes they can form (e.g., a cube or a cuboid).

Give children old glossy magazine paper. Ask: "Can you turn this into a cylinder shape?" If they roll up the paper and stick the edges together, it turns into a hollow cylinder. Let them make hollow cylinders of different diameters.

PAPER FOLDING

Let them feel their way into the activity. The more they work with paper, the greater will be their skill and accuracy in paper folding. Ask: "In how many ways can you fold a square paper into half?" They may fold a square vertically or horizontally or diagonally into half and make the edges match. Ask them to describe the shape that has formed after the paper is folded. Let them explore other ways of folding a square paper. "What shape do you get when you make a fold in one corner?" "How does the shape change when you fold one more corner?" "What happens when you fold all four corners?"

Now pose the same question about a rectangle. They may try to do the same with a rectangle and find that if they fold a rectangle from one corner to the opposite corner the edges do not get aligned, unlike a square. Here too they can experiment with folding different corners to see the shapes that emerge.

Similarly they may see that a triangle can only be folded in some ways to get a half triangle. They may also notice that different types of triangles fold differently. It is important that all these are discovered by children through play and occasional suggestions from the teacher (when the children seem to have run out of ideas).

Let children try to make regular shapes by creasing rectangular pieces of paper.

Tangrams

Designs can now be given in scaled down size. Children may be able to build the shapes by looking at the designs. The teacher can also give designs which use only a few of the pieces.

The teacher can also ask questions which require children to use the tangram pieces to create other regular shapes. Ask: "Take two small triangles. Can you make a square with them? Can you make a bigger triangle? Can you make any other shape?"

Figure 11

POSITIONS AND MAPS

Positional instructions: Introduce relevant vocabulary like left, right, turn, etc. Give children a sequence of instructions which they need to follow, say to go from one place to another place in the school. Use simple instructions: left, right, straight, turn around, etc.

Let them also describe a sequence of their movements using correct positional language.

Simple maps with concrete objects: Let children create a simple map of their class room by using rectangles, squares and circles to represent the teachers' table, their benches, perhaps a shelf or a dustbin. The teacher can discuss positional relationships, what is to the left or to the right, what is close to the teacher's table, what is further away etc.

This can be followed by simple route related problems. They can explore ways of getting from point S ('start') to point F ('finish') in a 3 by 3 grid as shown in Figure 12.

How many ways can you get from S to F if only vertical/horizontal moves are allowed? Which route is the longest? Which one is the shortest?

Game: Visualising shapes

The first student traces a regular shape like a square, rectangle or circle with his finger on the back of the second student who then feels the tracing, follows the turns, counts the sides and so on to visualise and name the shape.

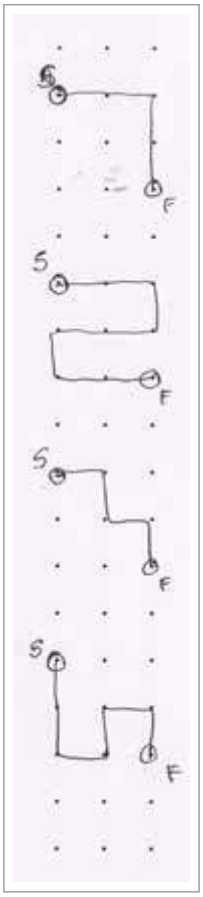


Figure 12

CLASS FOUR

At this point one can plan activities to get them ready to start formal geometry in the fifth year. The concept of angle and measurement of angle is one chief area where children face problems. The teacher should do plenty of activities involving measurements of corners of varied objects in the class using a square paper as a measure and many activities involving turn (using arms of the body, or arms of the clock).

3-D shapes

Let them study various 3-D objects (cubes, cuboids, cylinders, cones, triangular prism, spheres) by identifying the number of faces, shapes of the faces, the number of edges and the number of corners of each solid shape, and recording them in a table form.

NETS

Let children use nets of cubes and cuboids to build them. The teacher can draw the nets on the board and children can draw them using a square shape or a rectangular shape for making the net (Figure 13). At this stage they will not be able to make an accurate drawing using a scale.

Drawing 3-D shapes on triangular dot paper

Show them how to draw a cube on dot paper (Figure 14). They can also try drawing cuboid, sequence of connected cubes (towers) etc.

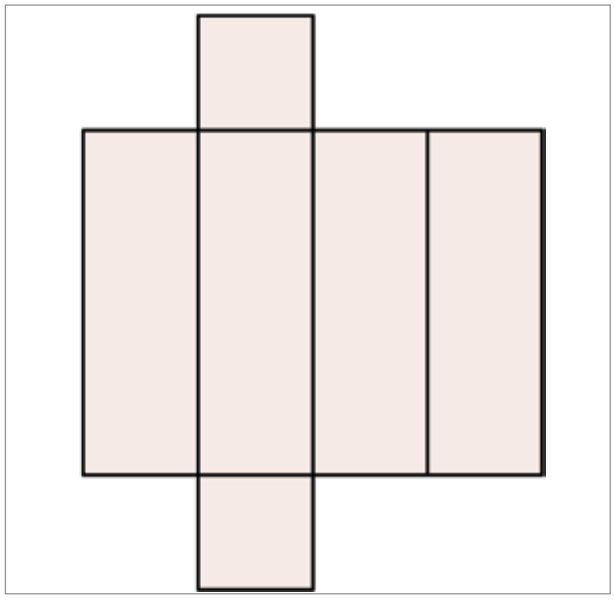


Figure 13

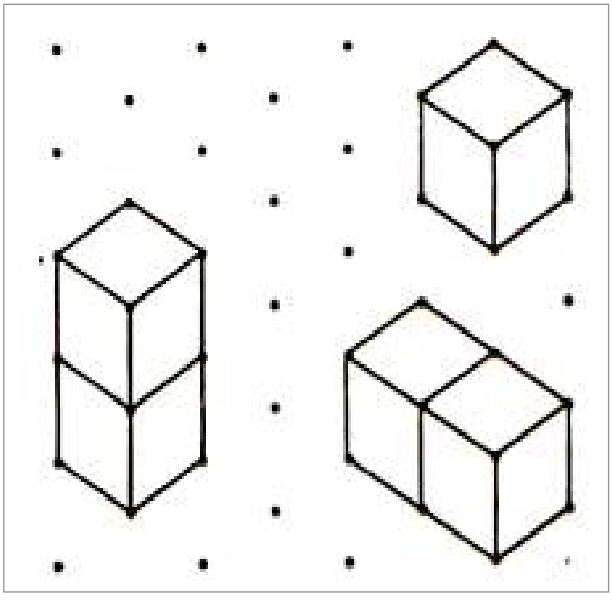


Figure 14

GEO-BOARD ACTIVITIES

Let them build composite shapes which fit together and describe them in terms of corners and lengths of the sides etc.

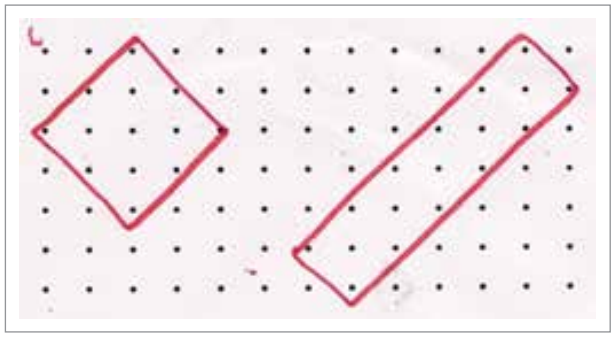


Figure 15

Now require them to build regular shapes linking pegs diagonally as well. Ask them to create a square or a rectangle (Figure 15). To get the sides of a rectangle or a square straight and parallel involves greater challenge now. It needs finer observation skills.

Follow up geoboard activities with dot paper based explorations.

Suggested explorations

Use a 3 by 3 dot grid and connect the dots with straight lines (horizontal or vertical or slant) to make different shapes (see Figure 16).

How many different shapes can you make? Name some of them.

Which shape has the maximum number of edges?

What is the minimum number of dots needed to make a shape? What shape was this? What is the maximum number of dots that you used? What shape did you get?

Could you make squares of different sizes? Could you make rectangles of different sizes? How many types? Could you make triangles of various sizes? How many types?

2-D shapes

Materials: Hexagons, large squares, small squares, different types of triangles, circles, pentagons and octagons.

Figure 17

Let children use some shapes to create symmetric designs and closed patterns as shown in the picture (Figure 17).

"What shapes can you make with two triangles and with three triangles?" Draw as many of them as you can on a sheet of paper (draw the outlines of the composite shapes formed).

"Can you make a bigger triangle using the small triangles?" (The bigger triangle need not be of the same shape as the smaller ones.)

"Can you make a triangle using square shapes? Why or why not? Can you make a square shape using triangles? Why or why not?"

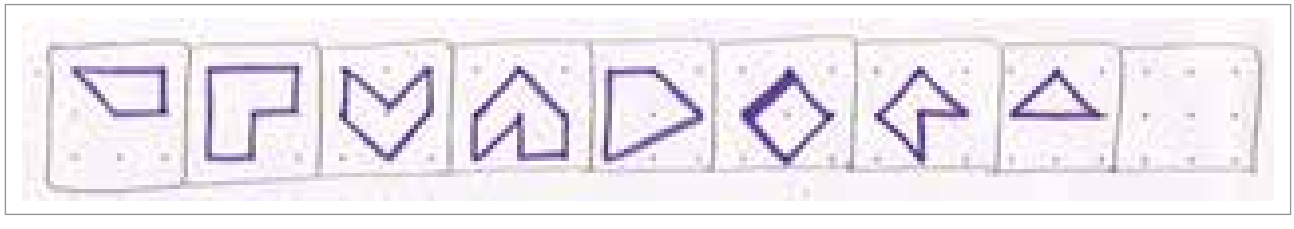


Figure 16

Outlining activities with shapes

Children can trace the outline of a shape on plain paper and check the number of ways the shape can fit into its own outline.

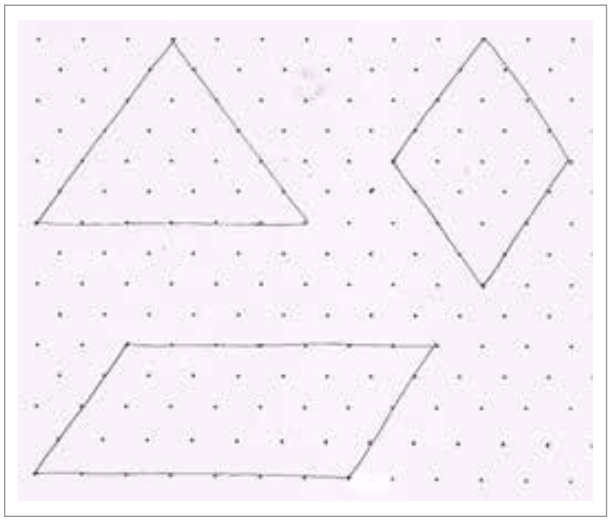


Figure 18

Spotting shapes in other shapes: Draw some shapes as shown in Figure 18.

Ask: "Can you see a hexagon and some triangles in these shapes?"

Tessellations (tiling patterns)

Children can be shown tessellating patterns found on paths and walls, or pictures of tessellations. They can play around with various shapes to create patterns which tessellate. Let them make a table of shapes that tessellate and shapes that do not. Through experimentation they can find shapes that fit together and shapes that do not fit together. They can also make tessellations on dot paper.

Ask: "Which shapes cover a surface without leaving any gap?" "Which shapes leave a gap?" "What is the shape of that gap?"

PAPER FOLDING

Start with a square sheet of paper. Ask the children to show you that the given shape is actually a square. They will demonstrate by folding one edge onto the opposite edge (horizontal fold) and one edge onto the adjacent edge (diagonal fold). They can make a quarter fold to demonstrate that all the angles are equal.

Let them explore folding a paper (square or rectangular) in different ways. Some possible ways can be to fold a paper at the centre horizontally but not at the centre while folding vertically. What shapes are formed now? Let them make a border fold on all four sides of a paper. Open and see the shapes that have formed. Let them try different slant folds which can result in a pointed roof shape or a boat shape.

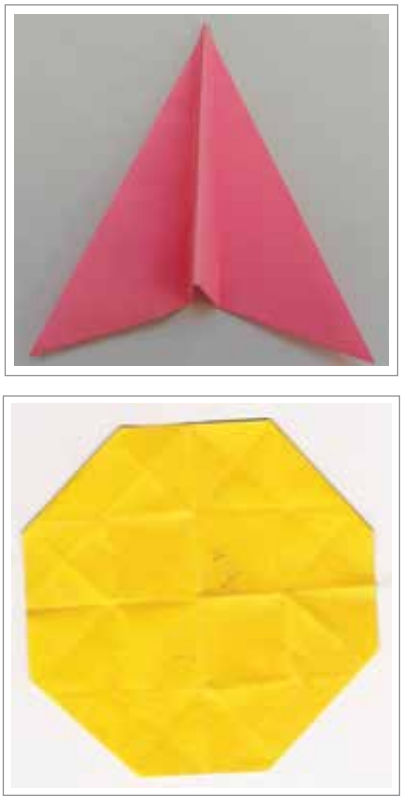


Figure 19

One can also pose a question the reverse way. Show them a sheet you have already folded (not involving more than two or three steps) and ask them to figure out the folds needed to create that shape.

Creating different shapes using square paper: Show children a parallelogram, a rhombus, a hexagon and a pentagon and ask them if they can make these shapes out of a square sheet of paper. They can experiment with folding the corners to create these shapes.

Folding a circular sheet of paper (e.g., paper plates) generates interesting shapes and designs.

They can try to create a square, a rectangle and a triangle. Also show the children a rhombus and a hexagonal shape and ask them if they can fold the sheet to get those shapes.

Angle as a corner

Let children use a square piece of paper to check the corners of different shapes in the class (teacher's table, floor tiles, text books, triangle pieces, other polygon shapes).

They can make a list of the shapes in the class, those which are square corners, those which are less-than-square corners and those which are more-than-square corners. Teacher may decide to use formal vocabulary: right angle, acute angle and obtuse angle.

Game: "What is my shape?"

Materials: Set of different shapes in pairs (rectangles, squares, triangles of different shapes, parallelograms, rhombi, trapeziums, pentagons, hexagons, circles).

Teacher separates out the two sets, keeps one set and gives the other set to the children. One of the children picks up any one shape from the teacher's set.

The child now describes the shape in terms of its corners alone to the other children. (Ex: My shape has one square corner and two corners less than a square.)

The child may also give information about sides and corners. (Ex: I have 4 square corners and 4 sides, but I am not a square.) Or she may describe the slant nature of the edges. (Ex: I have 4 sides and two of my opposite sides are slanted.)

The other children must now deduce what shape it could be and thus pick up its twin from the other set.

This activity trains the children to look for special properties of the shapes. They soon realise that if they do not describe a special property there can be many shapes with that description.

I have found this game to be useful in sharpening children's observation and logical skills.

Angle as a turn

Connect two sticks with a rubber tube piece so that it forms a rigid bendable L shape.

Let them fix one stick firmly and turn the other around noting the angle between the sticks as they make a quarter turn, half turn, three-quarter turn and full turn.

This activity can also be used for comparing two angles. They can place it over one angle by aligning the arms of the L shape with the arms of the first angle. They can then place it over the second angle and find which angle is the bigger one.

Students can be given a worksheet with many pairs of equal angles drawn in a jumbled up manner. They can then identify pairs of equal angles using the above approach.

Angle hunt

Children can be asked to look for acute angles in their surroundings. Let them examine chairs, clocks, hinges, fan blades, etc.

Lines

Through paper folding activities one can discuss lines, points.

Ask children to fold a paper in such a way that many lines pass through one point. The point can coincide with a corner of the square, it can lie on one side of the square, or it can lie in the interior of the square.

The concept of parallel lines and perpendicular lines can be brought in.

Lines on dot paper

On a sheet of dot paper, circle two random points which are at a distance from one another. Ask: "If these two points are connected with a straight line, which other points will lie on the line?" Circle three random points. Ask: "Will all these lie on one line?"

Line drawings

Line drawing is a very interesting activity. It helps in developing the skill of using a scale, aligning it and marking points exactly. It requires them to identify pairs of corresponding points. It can be done initially on dot paper and later on plain paper.

Steps for construction

Draw a pair of straight lines of length 10 cm at right angles to each other, as shown in Figure 20. The point where they meet is numbered 0. The vertical line is numbered from top to bottom, 10 to 0. The horizontal line is numbered from left to right, 0 to 10. Now join 10 on the vertical line to 1 on the horizontal line, 9 on the vertical line to 2 on the horizontal line, 8 to 3, 7 to 4, and so on. This activity produces a pleasing curved line effect.

Once children learn this technique, they generally come up with various ways of drawing the initial lines (V shape, triangular shape, 'plus sign' shape, etc) to create beautiful designs.

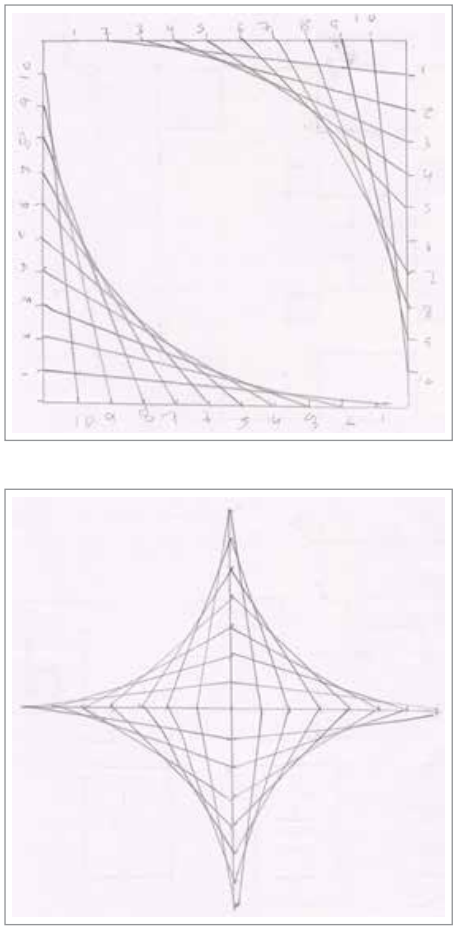


Figure 20

Shape dissections

Cut a large cardboard square into 3 to 5 pieces like a jigsaw puzzle. Let children put them back together to form a square.

This can also be tried with a rectangle, triangle or a circle.

Challenge

Draw a rectangle on a card board. Draw the diagonals and cut it along the diagonals to create 2 pairs of identical triangles. These 4 triangles can be rearranged in different ways to generate outlines of shapes. Let the children figure out the way the four triangles fit on the outline.

Map

Use maps of the neighbourhood of the school for children to map the route from home to school (provided it is not too far). Let them describe the route using positional language.

Point to a place in the map and ask: "If you are standing by this building, is the bank to your left?" "What building lies at the first right?" "Do you turn left or right at this point?" "How will you describe the route to the bus stand?"

Blindfold Game

Another fun activity to build positional sense in children is to get children to do a blindfold walk in a room. One child gives instructions to the other who is blindfolded in the form 'Take two steps forward, turn right, take three steps, now turn left' etc. The purpose of the game is to reach from a start point to an end point, avoiding some obstacles in the room.

Many children's magazines bring out attractive puzzles like mazes, matchstick puzzles, route problems, spotting hidden cubes in a stack of cubes, etc. All these assist in building spatial and logical skills.

Figure 20

Padmapriya Shirali

Padmapriya Shirali is part of the Community Math Centre based in Sahyadri School (Pune) and Rishi Valley (AP), where she has worked since 1983, teaching a variety of subjects – mathematics, computer applications, geography, economics, environmental studies and Telugu. For the past few years she has been involved in teacher outreach work. At present she is working with the SCERT (AP) on curricular reform and primary level math textbooks. In the 1990s, she worked closely with the late Shri P K Srinivasan, famed mathematics educator from Chennai. She was part of the team that created the multigrade elementary learning programme of the Rishi Valley Rural Centre, known as 'School in a Box'. Padmapriya may be contacted at padmapriya.shirali@gmail.com