11 Ancient India and Mathematics

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One way to approach this very broad topic is to list all that the ancient Indian mathematicians did – and they did do an enormous amount of Mathematics: arithmetic (including the creation of decimal place notation, the invention of zero), trigonometry (the detailed tables of sines), algebra (binomial theorem, solving quadratic equations), astronomy and astrology (detailed numerical calculations). Later Indian Mathematics, the Kerala school, discovered the notions of infinite series, limits and analysis which are the precursors to calculus.

Since these details are easily available I am not going to list them here. What is of interest to me is to understand in what sense these activities were 'mathematical'. By doing so, I am also responding to the charge that these people were not doing Mathematics but something else. This is a charge similar to that addressed to Science in ancient India – the claim here is that what was being done in metallurgy, for example, was not Science but only craftsmanship. Similarly, there is a claim that Indian Mathematics is not really Mathematics since it was not axiomatic, it was related to the world whether in calculation of planet positions or dimensions of the sacrificial pyre, it was not really logic since it was explicitly related to the empirical and so on.

Indian Mathematics was explicitly engaged with the natural world and is in some sense grounded upon the nature of our cognition as well as the nature of the world. It was more about doing and in a sense closer to the constructivist paradigm. A famous example is the Indian mathematicians' pragmatic acceptance of square root of 2 (as something that is used in construction, for example) as against its rejection by the Pythagoreans on idealistic grounds.

Another uniqueness of Indian Mathematics was the form in which it was written. Early Mathematics was often written in poetic form. While it would seem as if the Indian Mathematicians did not use symbols like we see in modern texts, this is not completely true since they used alphabets of Sanskrit to stand for numbers. The implications of writing Mathematics in a poetic form have not been considered in detail and suffice it for me to say here that this approach has important implications for Mathematics education!

There are some things in common between Indian and Greek Mathematics but there are also significant differences – not just in style but also in the larger world view (which influences, for example, the completely different ways of understanding the nature of numbers in the Greek and the Indian traditions). This difference has led many writers to claim that Indians (and Chinese among others) did not possess the notions of Science and Mathematics. The first, and enduring response, to the question of Science and Mathematics in ancient non-Western civilizations is one of skepticism. Did the Indians and Chinese really have Science and Mathematics as we call it now? This skepticism has been held over centuries and by the most prominent thinkers of the west (and is in fact so widespread as to include claims that Indians did not 'have' philosophy, logic and even religion). So even before we begin to understand the nature of Science and Mathematics in ancient India we need to have a response to this skepticism.

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One type of response is to consider the development of the ideas of Science and Mathematics in the west. The west did not have disciplines called the sciences until a few centuries ago. What they had were a variety of disciplines such as physics, chemistry, metallurgy, geology and so on. In the early eighteenth century these disciplines began to get unified under the name of 'Science' and debates during this time illustrate how problematic this unification was since it was very difficult to find common elements in all these disparate disciplines. (In fact, the word 'scientist' was itself coined by Whewell in 1833 and prominent scientists during this time repeatedly wondered as to what was common to all these disciplines.) This debate on what constitutes a science continues even today, and is best manifested in the scientists' reaction (mostly negative!) to calling social science as a science or worse, astrology as science.

There is indeed a genuine problem in placing disparate disciplines such as physics, chemistry, geology, immunology and so on under one category called science. There is little that is common in the practices of these disciplines as well as in the subject matter. This problem leads us to search for a common methodology; something which could be called as the 'scientific method' and which presumably would be found in everything we call science. But the search for this elusive methodology has been long and difficult, and not entirely successful. It has often been simplified to say that scientific methodology is based on the activities of theory and experiment but such a rendering also makes many other human activities scientific.

One way we can understand the nature of science is by viewing it as a title; a title given by a group of people who see themselves as representatives of science. In fact, if we see how national associations of science talk about science we can clearly see this attempt – by these groups to regulate what is science and what is not – as an indication that science is primarily a title.

Given this background, it becomes more obvious that the question of whether Indians and Chinese "had" science and Mathematics is actually a question that can be reasonably asked only by (1) first understanding how different subjects came to be grouped under science or Mathematics and (2) as to why such a question is not posed in the western context. As is well-known, there is very little in common between Aristotle's science and modern science. On the contrary, it was the overthrow of Aristotle's ideas about the natural world that made possible science as we know it today. But in spite of this, we often see scientists talking about Greek science without qualifications but when it comes to science in other cultures – whether ancient or modern – there is often deep skepticism.

The case of Mathematics is slightly different from science although similar questions about the unification of different disciplines remain. That Mathematics was a Greek invention and that it was one of the most influential disciplines which catalyzed other disciplines such as logic has been accepted for a very long time and is still very much a part of 'cultural pedagogy'. (Even today, very influential textbooks, specialized books as well as popular ones continue this myth as if other cultures had no access to these 'subjects'). However, unlike science, there seems to have been less of a confusion about what defines Mathematics. In the case of science, the disciplines came first and then they were put under the category of science. In Mathematics, the situation was quite different since right from the beginning certain kinds of activities were seen to belong to the Mathematical. And this was true for both Greek and Indian traditions.

But the question that is so problematic for science is also partly true for Mathematics. How do we recognize new disciplines such as calculus, differential equations etc. as belonging to Mathematics in the same way that arithmetic and geometry were Mathematics? If geometry is a paradigm example of Mathematics for Euclid, then what is common to the axiomatic system of Euclid and the various new ideas in calculus, topology and other disciplines which are placed under Mathematics? For example, when calculus was created it was not like the Euclidean axiomatic system. Then why is calculus called Mathematics in the same way that Euclidean geometry is Mathematics?

In general, though, it is easier to identify Mathematics in comparison to science. For example, the objects with which Mathematics deals with are very special ones such as numbers, sets, functions and matrices. There is, in general, some commonality in the 'objects of discourse' of Mathematics unlike science since physics deals with the physical world (remember Newton's belief that a primary task of physics was to distinguish real motion from apparent motion), chemistry with organic and inorganic molecules (much of which are synthesized and created in the laboratory), biology with living organisms. In the case of Mathematics, set theory has overlap with arithmetic and algebra, topology with set theory and so on. There is more coherence in the Mathematical objects that occur in these various disciplines.

There are also other common indicators in the different sub-disciplines of Mathematics: the role of operators, the activity of calculation, the creative use of symbols, the creation of new kinds of symbols, the fundamental and essential role of the equality sign (and related to it the inequalities). Most of these characteristics are also closely linked to a very specific way of dealing with language (specifically, semiotics). Thus, Mathematics as a particular kind of 'language' is another common theme that links these various sub-disciplines of Mathematics. These are characteristics which are common to the many sub-disciplines of Mathematics.

They are also common to ancient Indian Mathematics, whether in the fields of arithmetic, trigonometry, algebra or analysis. But discovering these commonalities should not blind us to the unique differences which characterize the cultural imagination inherent in Mathematics. If we take this point seriously, then we might see more clearly that for the ancient Indian practitioners there is no clear distinction (in contrast to the Greeks and later on the western intellectual traditions) between science and Mathematics, just as there is little difference between science and logic. This also leads the Indians and the Greeks to have differing views on the nature of mathematical truth and mathematical objects.

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Some Great Indian Mathematicians

1. **Lagadha (c 1300 B.C):** The earliest mathematician to whom definite teaching can be ascribed to, and who used geometry and elementary trigonometry for his astronomy.

2. **Baudhayana (c 800 B.C):** He is noted as the author of the earliest Sulba Sutra which contained several important mathematical results; the now known Pythagorean theorem is believed to have been invented by him.

3. **Yajnavalkya (c 800 B.C):** He lived around the same time as Baudhayana and is credited with the then-best approximation to π.

4. **Apastamba (c 500 B.C):** He lived slightly before Pythagoras, did work in geometry, advanced arithmetic, and may have proved the Pythagorean Theorem. He used an excellent approximation for the square root of 2 (577/408, one of the continued fraction approximants).

5. **Aryabhatta (476-550 C.E):** His most famous accomplishment was the Aryabhatta Algorithm (connected to continued fractions) for solving Diophantine equations. The place-value system was clearly in place in his work and the knowledge of zero was implicit in Aryabhata's place-value system as a place holder for the powers of ten with null coefficients.

6. **Daivajna Varāhamihira (505-587 C.E):** His knowledge of Western astronomy was thorough. In 5 sections, his monumental work progresses through native Indian astronomy and culminates in 2 treatises on Western astronomy, showing calculations based on Greek and Alexandrian reckoning and even giving complete Ptolemaic mathematical charts and tables.

7. **Brahmagupta 'Bhillamalacarya' (589-668 C.E)**: His textbook Brahmasphutasiddhanta is sometimes considered the first textbook "to treat zero as a number in its own right." Several theorems bear his name, including the formula for the area of a cyclic quadrilateral: 16 A2 = (a+b+c-d)(a+b-c+d)(a-b+c+d)(-a+b+c+d).

8. **Bháscara (c 600 – c 680 C.E)**: He was apparently the first to write numbers in the Hindu-Arabic decimal system with a circle for the zero, and who gave a unique and remarkable rational approximation of the sine function in his commentary on Aryabhata's work. Bhaskara's probably most important mathematical contribution concerns the representation of numbers in a positional system.

9. **Mahavira (9th-century A.D)**: He is highly respected among Indian Mathematicians, because of his establishment of terminology for concepts such as equilateral, and isosceles triangle; rhombus; circle and semicircle. He asserted that the square root of a negative number did not exist and gave the sum of a series whose terms are squares of an arithmetical progression and empirical rules for area and perimeter of an ellipse.

10. **Sridhara (c. 870 – c. 930 C.E)**: He wrote on practical applications of algebra and was one of the first to give a formula for solving quadratic equations and gave a good rule for finding the volume of a sphere.

11. **Bháscara Áchárya / Bhaskara II (c 1114-1185 C.E)**: His "Chakravala method," an early application of mathematical induction to solve 2nd-order equations, has been called "the finest thing achieved in the theory of numbers before Lagrange." He conceived the modern mathematical convention that when a finite number is divided by zero, the result is infinity.

12. **Madhava of Sangamagrama (1340-1425 C.E)**: He did work with continued fractions, trigonometry, and geometry. Madhava is most famous for his work with Taylor series, discovering identities like sin q = q - q3/3! + q5/5! - ... formulae for π, including the one attributed to Leibniz, and the then-best known approximation $\pi \approx 104348 / 33215$.

13. **Srinivasa Ramanujan Iyengar (1887-1920 C.E)**: He produced 4000 theorems or conjectures in number theory, algebra, and combinatorics. Because of its fast convergence, formula of an odd-looking Ramanujan is often used to calculate

π: 992 / π = √8 ∑k=0,∞ (4k! (1103+26390 k) / (k!4 3964k))

14. **Prasanta Chandra Mahalanobis (1893-1972 C.E)**: He is best remembered for the Mahalanobis distance, a statistical measure. He made pioneering studies in anthropometry in India. He contributed to the design of large scale sample surveys

15. **Satyendra Nath Bose (1894-1974)**: As an Indian physicist, specializing in mathematical physics, he is best known for his work on quantum mechanics in the early 1920s, providing the foundation for Bose-Einstein statistics and the theory of the Bose-Einstein condensate

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